NETWORKS

NOTES

GATE 2009

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Network Theory

May 30, 2007.

1. Basics
2. Theorems
3. L.T.
4. Transients < dc, ac
5. Ac Analysis

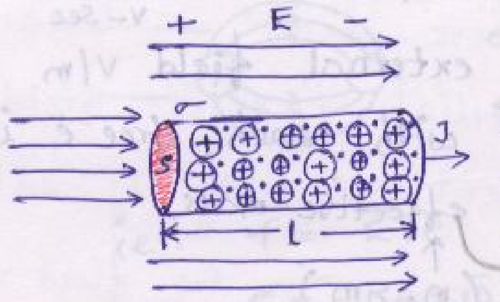
Ref:

1. Network Analysis – Van valkenburg
2. ✓ Engg. circuit analysis – Hayt & kemmerly
3. previous papers : GK pub.
   (i). GATE < EE, EC, INSTRUMENTAL (1990-2007)
   (ii). IES < EE, EC
   (iii). IAS – Prelims – EE (1994-2006) (Raj kamal text book) (is central kali)

UPSC

Basics.

→ The mechanism of energy flow through the conductor and ohm's law :-



⊕ ⇒ Ag^+ ion, immobile, Larger in size ie 10^3 times than $\bar{e}$.

• ⇒ free e^-

→ $Ag^{+1} \longrightarrow +1$

$Cu^{+2} \longrightarrow +2$

$Au^{+2} \longrightarrow +2$

$Al^{+3} \longrightarrow +3$

→ The mobility of free $\bar{e}$'s in a Ag, is several times to that of other conductors so its conductivity is very high.

→ Generally in any conductor, there are 10^{18} to 10^{23} atoms per unit volume (ie per unit cube) and hence there are 10^{18} to 10^{23} free $\bar{e}$s per unit volume in a Ag conductor. ie every conductor is a very rich of free $\bar{e}$s.

→ In the presence of external field different free e^- will under go diff. forces [due to a large no. of free $\bar{e}$s] and hence they will move with diff. velocity. But only one velocity is defined, so called drift velocity. It is an avg. velocity of all the free $\bar{e}$s within a conductor. and is given by $v_d = \mu E$ m/s.

μ = mobility of free $\bar{e}$s $\frac{m^2}{V\text{-}sec}$

E – Applied external field V/m

→ The K.E. associated with each free $\bar{e}$ is

$KE = \frac{1}{2} m_e v_d^2$ J

$m = 9.11 \times 10^{-31}$ kg

effective mass ($m_e \approx m$)

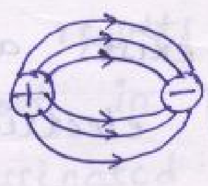
m_e is the mass of free $\bar{e}$ while it is in a motion.

The first Half of the Ohm's experiment when the conductor not carrying electrical energy $E=0$:-

→ when $E=0 \Rightarrow v_d=0 \Rightarrow K.E.=0$
ie all the free $\bar{e}$ are in the rest.

→ Since the conductor is operating at room temp. (27°C or 300°K), diff. free $\bar{e}$'s will acquire diff. thermal energies [due to a Large no. of free $\bar{e}$] and hence they will move in diff. directions in a random manner the net flow of $\bar{e}$ motion, in any direction zero, ie the charge motion is zero and the i is zero and also the current density [J] is zero.
ie when $E=0$, then $J=0$.

Second Half of Ohm's experiment, when the conductor is carrying electrical energy [$E \neq 0$] :-



when the conductor is subjected to an axial electric field, the force will be exerted on every free $\bar{e}$.

ie. $\vec{F} = \vec{E}.e$ N
$e = -1.6 \times 10^{-19}$ C

Since 'e' is -ve, there exists the direction of force is in opposite to that of E. and hence there exists a net $\bar{e}$ motion ie the charge motion in the direction opposite so that of $\bar{E}$.

The magnitude of charge is given by

$q = ne \quad C$, n = no. of free $\bar{e}$'s crossing a reference cs area, a variable quantity due a large no. of free $\bar{e}$.

$e = -1.6 \times 10^{-19} \; C$

→ The time rate of flow of electric charges is nothing but the electric i ie

$$i = \frac{dq}{dt} \quad A$$

Since q is -ve, the conventional current direction is opposite that of the charge motion ie $\bar{e}$ motion [ie in the dire. of $\bar{E}$]

The current per unit cs area is nothing but the current density resulted within a conductor

ie $J = \frac{i}{s} \quad A/m^2$

Since 's' is a scalar, the dire. of 'J' is in the dire. of 'i', ie [in the dire. of E.

Acc. to Ohm, there exists a linear relation b/w the applied electric field and resulting current density by $J \propto E$

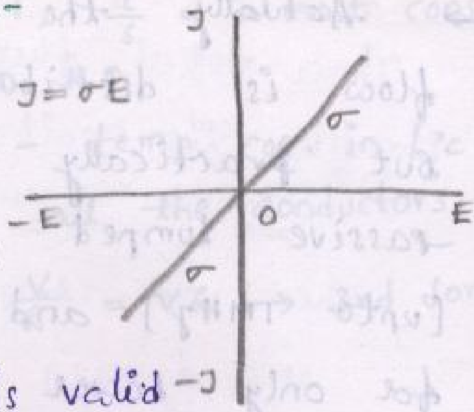
$J = \sigma E$ → Ohm's law in the field theory form.

σ → Conductivity of the conductor.

J – E characteristics :-

At the origin $J = \sigma E$

$E = 0 \Rightarrow J = 0$ and σ is not equal to zero.



Limitation:-

The ohm's law is valid only when proportionality const. σ is const. ie the temp. is kept constant condition.

At the const. E, as temp. increases from room temp. there exists an increase in collisions among the free $\bar{e}$'s and hence the mobility falls, so the conductivity decreases. [Here the collisions b/w the free $\bar{e}$'s and +ve ions are assumed to be const., since E is kept constant.].

At a const. TEMP. as 'E' increases there exists an increase in collisions b/w the free $\bar{e}$s and the +ve ions [larger in size], which results the fall lost in v_d and hence the lost in K.E. This losted energy will be dissipated in the form of heat, which results the volt. drop across the conductor. [Here the collisions amount, the free $\bar{e}$s are assumed to be const, since the temp. is kept const.

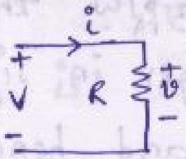
→ Actually the opposition for the energy flow is distributive through the conductor. But practically this is approximated into passive lumped R, L, C's for lower freq's [upto 1MHz] and hence n/w theory valid for only lower freq's.

At higher freq's we can't derive the lumped elements so no lumped electric n/w, so no n/w theory ie field theory is applicable.

field theory approach of solving the distributive electric n/w's. are valid for all freq's starting from zero [DC].

So the currents through all the 3 passive lumped elements will always flows from +ve to −ve terminals.

Resistance R :-



→ Since $J = \sigma E$

$\Rightarrow \frac{i}{s} = \sigma\left(\frac{v}{l}\right)$

$\Rightarrow v = \left(\frac{l}{\sigma s}\right) i$

$\Rightarrow v = Ri$ → Ohm's law in ckt theory form

$\therefore R = \frac{l}{\sigma s}$

Limitation :

The Ohm's law is valid when R is kept const ie temp. is kept const.

→ As $T\uparrow \Rightarrow l\uparrow, s\uparrow, \frac{l}{s}$ = almost const.

$\sigma\downarrow$ so $R\uparrow$

→ $R_t = R_o(1+\alpha t)$, α – temp. coe. in /°c, which is +ve for all the conductors.

→ Since $V = Ri \Rightarrow i = \frac{V}{R} = VG$ → 3rd form of ohm's law.

G = conductance ℧

Since $i = \frac{dq}{dt}$, $v = R.\frac{dq}{dt}$ → 4th form ohm's

→ $R = \frac{L}{\sigma S} \Rightarrow \sigma = \frac{L}{RS} = \frac{m}{\Omega - m^2} = ℧/m$ (or) S/m

→ Resistivity $\rho = \frac{1}{\sigma} = \frac{RS}{l} = \frac{\Omega - m^2}{m} = \Omega - m$

→ power $P = \frac{dw}{dt} = \frac{dw}{dq}\cdot\frac{dq}{dt}$

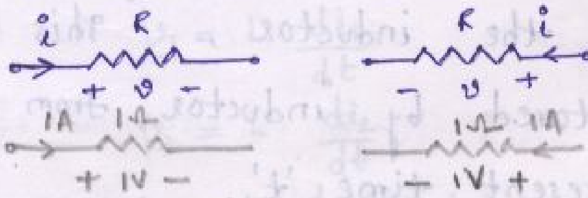
→ $P = i^2R = v^2/R$ (ω) $= v.i$ (ω)

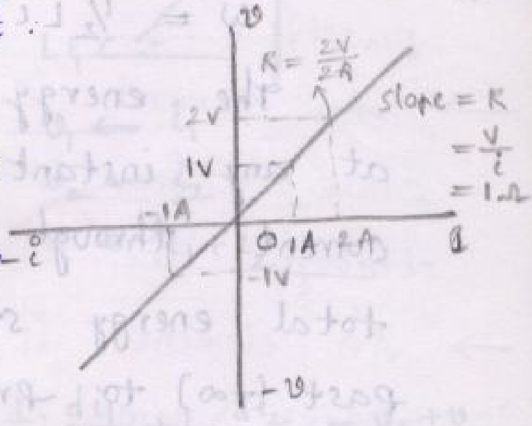
→ Energy $dw = p\,dt \Rightarrow w = \int p\,dt$ (J)

$w = \int i^2R\,dt = \int \frac{v^2}{R}\,dt$.

V–I characteristics:-

I Quadrant III Quadrant

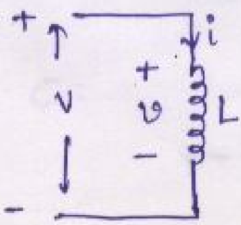




observations:-

1. Resistor is a linear, passive, bilateral and time invariant in V–I plane.

Inductance L:-



when a time varying i is flowing through the coil, a time varying magnetic flux will be produced. The total flux produced $\phi N = \psi$ (wb)

ϕ - flux per turn, N - no. of turns.

The total flux is proportional to the i through the coil ie $\psi \propto i$

$\Rightarrow \psi = Li$

The volt. drop across the coil is $v = \frac{d\psi}{dt}$

$$v = \frac{d}{dt}(Li) = L \cdot \frac{di}{dt}$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v \cdot dt$$

$-\infty \quad 0 \quad t \quad \infty$

power $p = vi = L \cdot \frac{di}{dt} \cdot i = Li \cdot \frac{di}{dt}$ (ω)

Energy $\omega = \int p\, dt$

$$= \int Li \cdot \left|\frac{di}{dt}\right| \cdot dt \quad (J)$$

$$p = Li\frac{di}{dt} = \frac{d}{dt}(\tfrac{1}{2}Li^2)$$

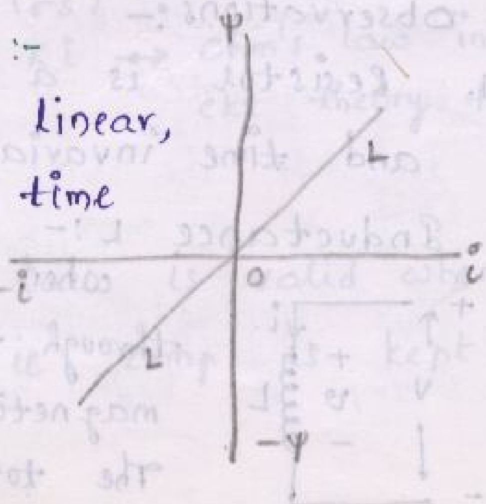
$$\omega = \int \frac{d}{dt}(\tfrac{1}{2}Li^2)\, dt$$

$$\omega = \tfrac{1}{2}Li^2 \; (J)$$

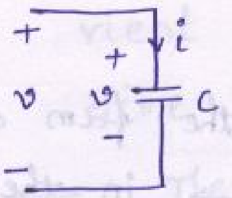
The energy stored in the inductor at any instant will depends only on the current through the inductor, This is total energy stored by inductor from infinite past $(-\infty)$ to present time 't'.

ψ - I characteristics :-

→ The inductor is a linear, passive, bilateral, time invariant element in ψ - I plane.



Capacitor C :-



$$i = \frac{dq}{dt}, \quad q \propto v$$

$$q = Cv$$

C - capacitor parameter.

$\rightarrow \quad i = C \cdot \frac{dv}{dt} \qquad i = \frac{d}{dt}(Cv)$

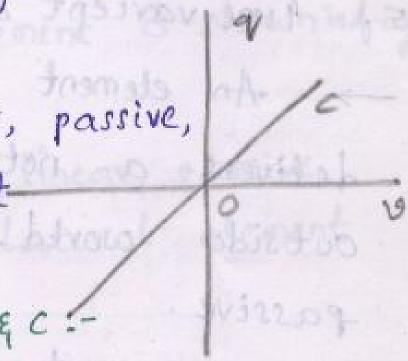
$\rightarrow \quad v = \frac{1}{C} \int_{-\infty}^{t} i\,dt$

$\rightarrow \quad p = C \cdot v \frac{dv}{dt} = \frac{C}{2} \cdot \frac{dv^2}{dt}$

$\rightarrow \quad \omega = \int \frac{d}{dt}\left(\frac{1}{2} C v^2\right) dt = \frac{1}{2} C v^2 \;(J)$

So energy stored in capacitor at any instant depends on voltage at that instant.

q-v characteristics :-



The capacitor is a linear, passive, bilateral, time invariant in q-v plane.

Relation b/w v & i in L & C :-

$L: \quad v = L \cdot \frac{di}{dt}$

$v_1 \leftarrow i_1$

$v_2 \leftarrow i_2$

$? \leftarrow i_1 + i_2$

$v_1 = L \cdot \frac{di_1}{dt}$

$v_2 = L \cdot \frac{di_2}{dt}$

$\therefore v = L \cdot \frac{d}{dt}(i_1 + i_2) = L \cdot \frac{di_1}{dt} + L \cdot \frac{di_2}{dt} = v_1 + v_2$

So the relation b/w v & i in L is linear and hence $v = L \cdot \frac{di}{dt} \rightarrow$ 5th form Ohm's law

$i = \frac{1}{L} \int_{-\infty}^{t} v\,dt \rightarrow$ 6th form Ohm's

$C: \quad i = C \cdot \frac{dv}{dt} \rightarrow$ 7th

$v = \frac{1}{C} \int_{-\infty}^{t} i\,dt \rightarrow$ 8th

NOTE :-

① . $W_L = \frac{1}{2} L i^2$ and $i = \int H \cdot dl$

② $W_C = \frac{1}{2} C v^2$ and $v = \int E \cdot dl$

so inductor stores energy in the form of magnetic field and capacitor → in the form of electric field.

Types of Elements :-

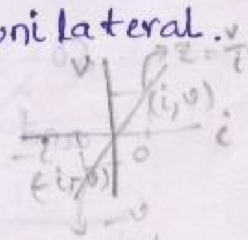
1. Active and passive
2. Linear and Non- linear
3. Bilateral and unilateral
4. Distributed and lumped
5. Time varient and invariant .

→ An element is said to be active if it delivers a net amount of energy to the outside world. otherwise it is said to be passive.

→ An element is said to be linear if its char.s for all time 't', is a st. line, through the origin, otherwise → Non linear

→ An element is said to be bilateral if it offers same impedance for either dire. of i flow, → otherwise → unilateral.

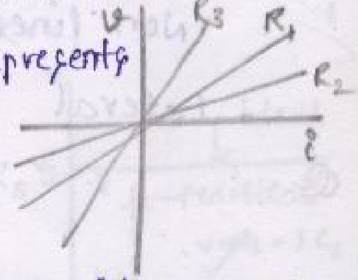
In other words for a bilateral element, if (i, v) is on the char.s then (-i, -v) must also be on the char.s.



$z = \frac{-v}{-i}$ Ω

→ An element is said to be time invarient if its char.s doesn't change with time otherwise → time varient.

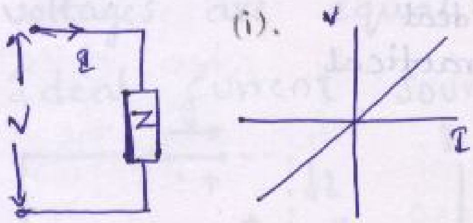
→ The besides char.s also represents passive, linear and bi-lateral.



NOTE:- The resistors, inductors, capacitors are passive if and only if $R \geq 0$, $L \geq 0$ & $C \geq 0$. Otherwise they are said to be active ie $R<0$, $L<0$ & $C<0$.

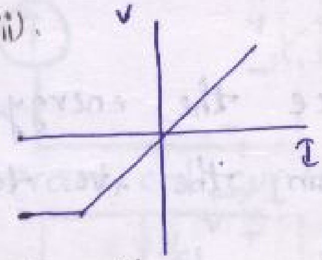
The v-I char.s of an element is shown in fig(b) then the element — ?

(i).



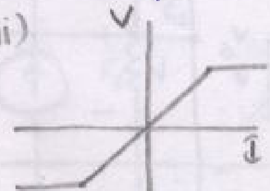
→ linear, passive, bilateral element.

(ii).



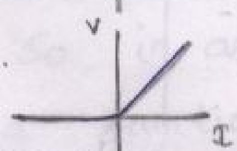
→ Non-linear, passive, uni-lateral element

(iii)



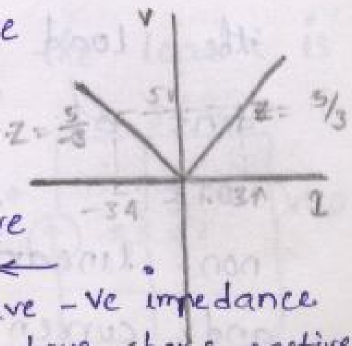
→ Non-linear, passive, Bi-lateral element

(iv).



→ Non-linear, passive unilateral.

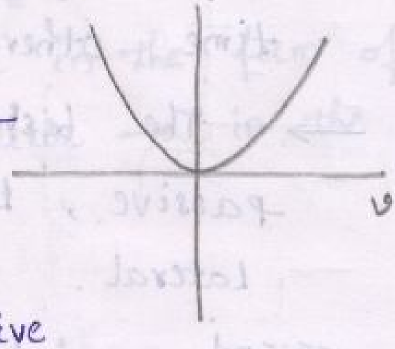
(v).



Non linear, active unilateral

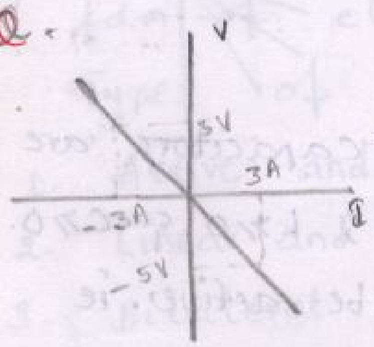
NOTE:- No passive element will have -ve impedance in any portion of its char.s. So above char.s → active

Q The voltage-current relations in a resistor $i = 2v^2$ then that element — ?



1 → Non linear, active, uni-lateral ←

Q.



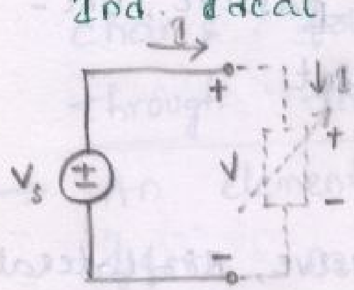
→ linear, active
bi-lateral $(i, v) = (-3, 5)$
$= (3, -5)$.

Obs :- All the linear elements are always bi-lateral and converse need not be true.

SOURCES :-

- independent
  - V
    - Ideal
    - practical
  - I
    - Ideal
    - practical
- dependent
  - VCVS
  - VCCS
  - CCVS
  - CCCS

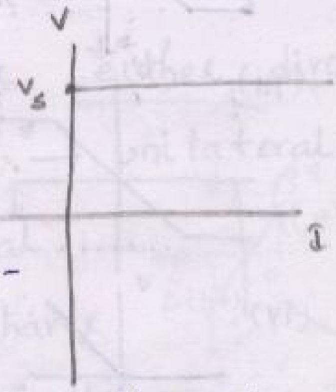
Ind. Ideal voltage sources :-



from any source the energy delivery is from the +ve terminal.

Source voltage $= V_s$

$V = V_s$ for all 'I'.

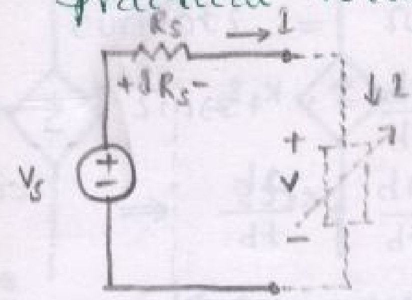


So in an ideal voltage source, the load voltage is independent of load i drawn.

NOTE :- All the sources are inherently non-linear in nature, since the voltage and current relation is non-linear.

They are basically active and unilateral elements.

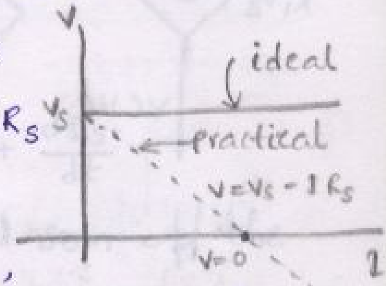
practical voltage source :-



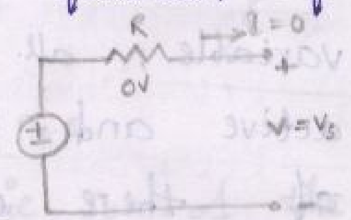
By writing KVL,

$V_s - I R_s - V = 0$

$\Rightarrow V = V_s - I R_s$



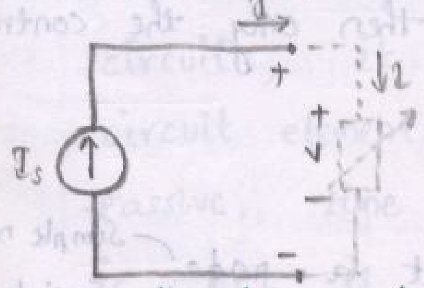
In a practical volt. source, the load voltage is a function of load i drawn.

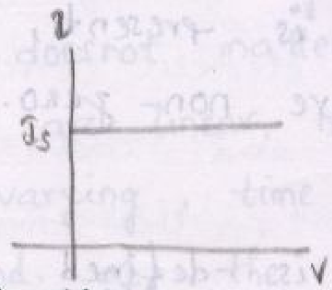


→ when $I = 0$, $V = V_s$.

ie when the i through any passive element is zero, then the two end voltages are equal and viceversa.

Ideal current source :-

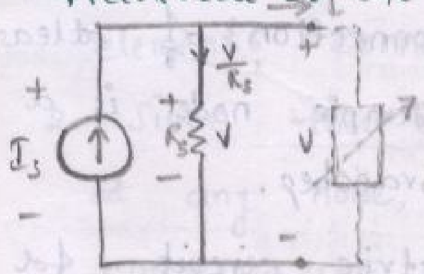


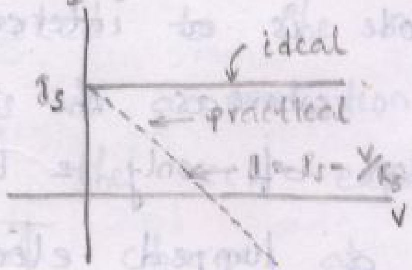


$I = I_s$ for all V.

In an ideal i source, the load V is ind. of i.

practical current source :-

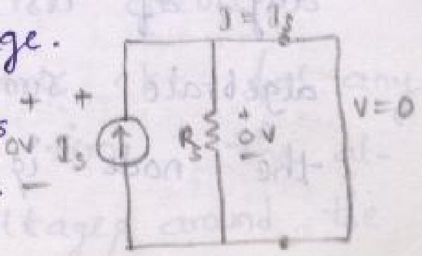




$-I_s + \frac{V}{R_s} + I = 0$

$\Rightarrow I = I_s - \frac{V}{R_s}$

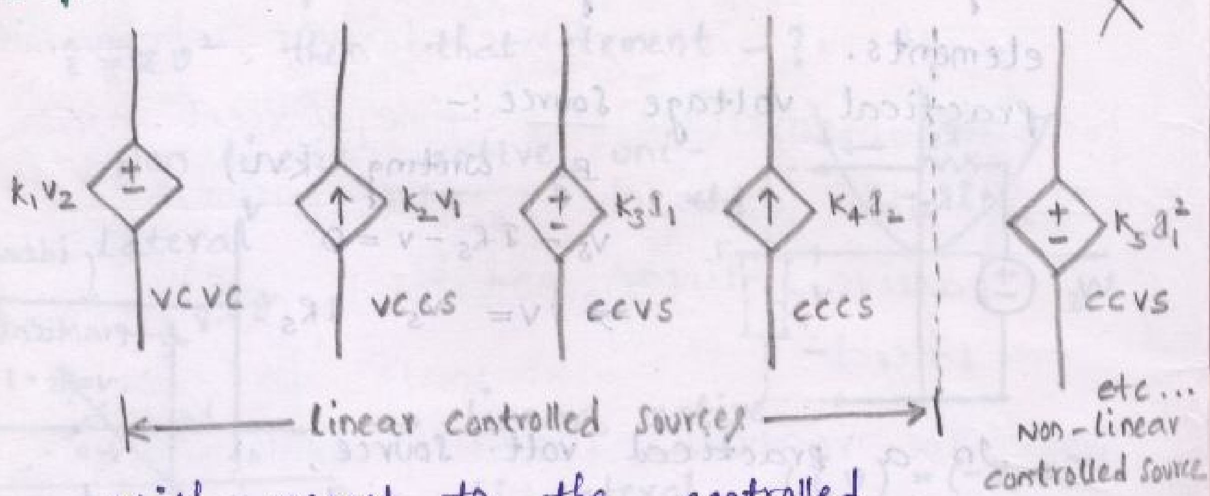
So in an ~~ideal~~ practical cs, the load i is a function of load voltage.



→ when $V = 0$ then $I = I_s$

ie the current always chooses a min. resistance path.

Dependent or controlled sources:-



With respect to the controlled variable, all controlled sources are linear, active and bi-lateral elements. The presence of these elements makes the n/w a linear active and bilateral.

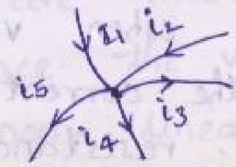
The controlled sources are said to be active elements ie the sources only when atleast one ind. source is present, then only the controlled var.s are non-zero.

K-laws:-

1. KCL:- It is defined at a node: simple node / principle node

principle node is a interconnection of atleast 3 branches, whereas the simple node is a interconnection of only 2 branches.

In a lumped electric circuit, for any of its nodes and at time 't', the algebraic sum of all the branch i's leaving the node is zero.



$-i_1 - i_2 + i_3 + i_4 + i_5 = 0$

$\Rightarrow i_1 + i_2 = i_3 + i_4 + i_5$ ie sum of entering currents = sum of the leaving currents.

$\rightarrow$ Since $i = \frac{dQ}{dt}$

$$\Rightarrow \frac{dQ_1}{dt} + \frac{dQ_2}{dt} = \frac{dQ_3}{dt} + \frac{dQ_4}{dt} + \frac{dQ_5}{dt}$$

$\Rightarrow Q_1 + Q_2 = Q_3 + Q_4 + Q_5$ ie sum of the entering charges = sum of the leaving charges.

$\rightarrow$ since $q = ne$,

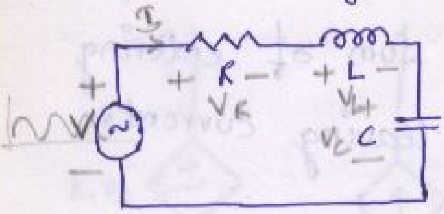
$$n_1 e + n_2 e = n_3 e + n_4 e + n_5 e$$

$\Rightarrow n_1 + n_2 = n_3 + n_4 + n_5$ ie sum of the entering e^-s = sum of the leaving $\bar{e}$s.

Features:-

1. The KCL applies to any lumped electric circuit, it doesnot matter whether the circuit elements are linear, non-linear, active, passive, time varying, time invarient etc. ie KCL is ind. of the nature of the elements connected to the node.
2. Since there is no accumulation of a charge at any node, the KCL expresses the conservation of charge at each and every node in a lumped electric circuit.

KVL :- In a lumped electric ckt for any of its loops at any of time, the algebraic sum of branch voltages around the loop is zero. $v - v_R - v_L - v_C = 0$



$\Rightarrow v = v_R + v_L + v_C$

$\Rightarrow$ since $v = \frac{\omega}{q} = \frac{\omega_R}{q} + \frac{\omega_L}{q} + \frac{\omega_c}{q}$

$\Rightarrow \omega = \omega_R + \omega_L + \omega_c$

features :-

1. The KVL is ind. of the nature of the elements, present in a loop.
2. KVL expresses the conservation of energy in a every loop of a lumped electric ckt.

→ KCL + Ohm's law = Nodal Analysis

KVL + Ohm's law = Mesh Analysis

since KCL & KVL are ind. each other, the nodal & mesh procedures are ind. to each other.

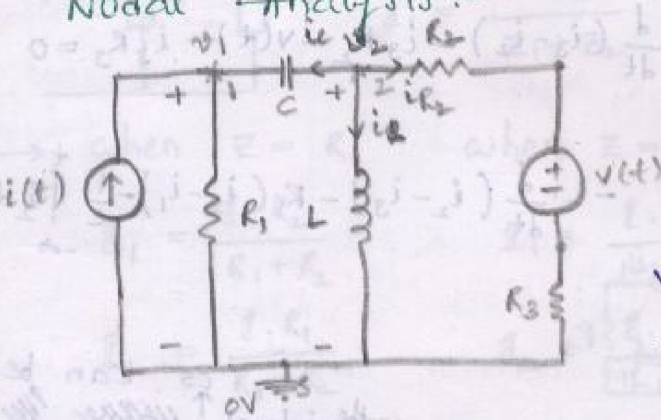
→ The above techniques are valid only for the lumped electric circuits, [where KCL, KVL are valid] and that to at a constant temp. [where the ohm's law is valid].

→ The K- laws are ind. of the nature of the elements, where as ohm's is a function of the nature of elements.

The ohm's law is defined across an element that element can be lumped or distributed $J = \sigma E$, where as the K- laws are applicable to only for the lumped electric circuits.

✓ The ohm's law is not applicable for active elements like sources, since the v-I relation is non-linear and it is applicable to only for the linear passive elements like R, L, C.

Nodal Analysis :-

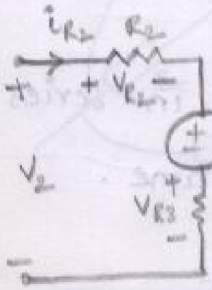


step 1 :-
1. Identify the no. of nodes.
2. Assign the node voltages with reference to ground node, whose voltage always = 0.
3. By using KCL first & ohm's next write nodal equations.

At Node 2 ; $\begin{bmatrix} V_2 > V_1 \\ V_2 > 0 \\ V_2 > v(t) \end{bmatrix}$ $i_c + i_L + i_{R_2} = 0$ (By KCL)

$$C \cdot \frac{d(V_2 - V_1)}{dt} + \frac{1}{L}\int_{-\infty}^{t} v_2\, dt + \frac{v_2 - v(t)}{R_2 + R_3} = 0$$

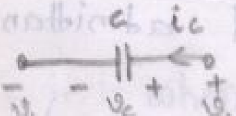
(By ohm's)



$$V_2 - V_{R_2} - v(t) - V_{R_3} = 0$$

$$V_2 - i_{R_2} R_2 - v(t) - i_{R_2} R_3 = 0$$

$$\Rightarrow i_{R_2} = \frac{v_2 - v(t)}{R_2 + R_3}$$



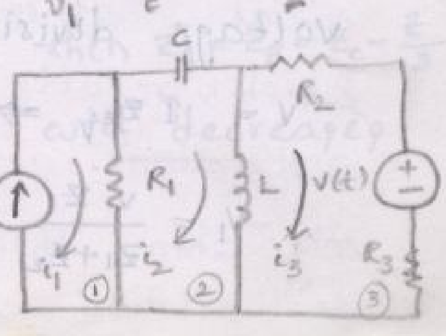
$$v_2 - v_c - v_1 = 0$$

$$\Rightarrow v_2 = V_1 + v_c \Rightarrow v_c = v_2 - v_1 \; ; \quad i_c = C\frac{dv_c}{dt} = C \cdot \frac{d}{dt}(v_2 - v_1)$$

At Node 1 :- $\begin{bmatrix} v_1 > V_2 \\ V_1 > 0 \end{bmatrix}$

$$-i(t) + \frac{v_1}{R_1} + C \cdot \frac{d}{dt}(v_1 - V_2) = 0$$

Mesh Analysis :-

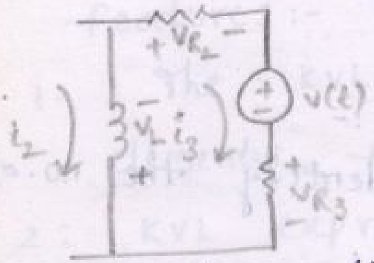


Steps :-

1. Identify the no. of meshes.

2. Assign mesh i's in clockwise.
3. By using KVL first and ohm's law next write the mesh equations.

Mesh 3:- $\begin{pmatrix} i_3 > i_2 \\ i_3 > i_1 \end{pmatrix}$

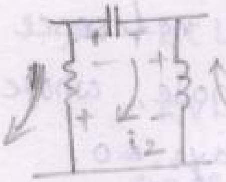


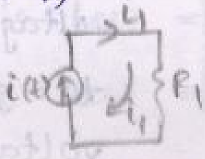
$$-V_L - V_{R_2} - v(t) - V_{R_3} = 0$$

$$-L \cdot \frac{d}{dt}(i_3 - i_2) - i_3 R_2 - v(t) - i_3 R_3 = 0$$

Mesh 2:- $(i_2 > i_3)$
$(i_2 > i_1)$

$$-L \cdot \frac{d}{dt}(i_2 - i_3) - R_1(i_2 - i_1) - \frac{1}{C}\int_{-\infty}^{t} i_2 \, dt = 0$$





Since the voltage across the ideal CS can be any value, it is not possible to write the mesh eq. for mesh 1.

KCL:-
$-i(t) + i_1 = 0$
$\Rightarrow i(t) = i_1$

Equivalent circuits:-

→ when 2 elements are said to be in series only the i through them are same.

for $\parallel^{el}$ → voltages are same.

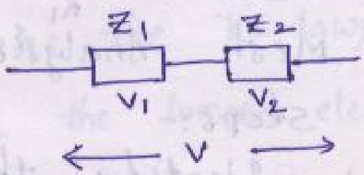
→ The impedances in series and admittances in $\parallel^{el}$ then we can add them.

$$Z_L = j\omega L \ \Omega \ ; \quad Z_c = \frac{1}{j\omega C} \ \Omega$$

voltage division principle:-

$$V = I Z_{eq} \Rightarrow I = \frac{V}{Z_{eq}}$$

$$\therefore V_1 = \frac{V \cdot Z_1}{Z_1 + Z_2} \ ; \quad V_2 = \frac{V \cdot Z_2}{Z_1 + Z_2}$$



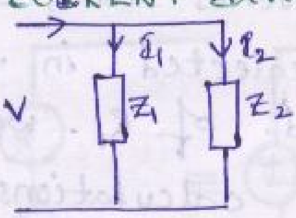
$Z_{eq} = Z_1 + Z_2$

→ when $Z = R$, when $Z = j\omega L$ when $Z = \frac{1}{j\omega C}$

$$V_1 = \frac{V \cdot R_1}{R_1 + R_2} \qquad V_1 = \frac{V \cdot L_1}{L_1 + L_2} \qquad V_1 = \frac{V C_2}{C_1 + C_2}$$

$$V_2 = \frac{V R_2}{R_1 + R_2} \qquad V_2 = \frac{V L_2}{L_1 + L_2} \qquad V_2 = \frac{V C_1}{C_1 + C_2}$$

CURRENT DIVISION :-



when taken as $Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$

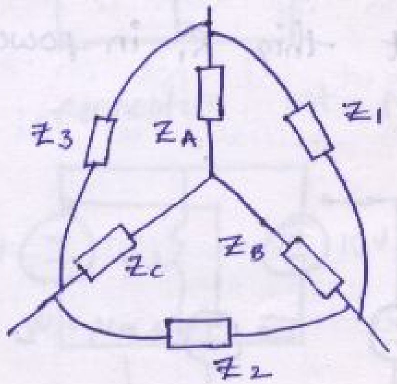
$$I_1 = \frac{I \cdot Z_2}{Z_1 + Z_2} ; \quad I_2 = \frac{I \cdot Z_1}{Z_1 + Z_2}$$

→ when $Z = R$, when $Z = j\omega L$ when $Z = \frac{1}{j\omega C}$

$$I_1 = \frac{I \cdot R_2}{R_1 + R_2} \qquad I_1 = \frac{I \cdot L_2}{L_1 + L_2} \qquad I_1 = \frac{I \cdot C_1}{C_1 + C_2}$$

$$I_2 = \frac{I \cdot R_1}{R_1 + R_2} \qquad I_2 = \frac{I \cdot L_1}{L_1 + L_2} \qquad I_2 = \frac{I \cdot C_2}{C_1 + C_2}$$

Y – Δ conversions:-



$$Z_1 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C} = Z_B + Z_A + \frac{Z_A Z_B}{Z_C}$$

$$Z_2 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A} \rightarrow Z_B + Z_C + \frac{Z_B Z_C}{Z_A}$$

$$Z_3 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B} \rightarrow Z_A + Z_C + \frac{Z_A Z_C}{Z_B}$$

$$Z_A = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$$

$$Z_B = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} ; \quad Z_C = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

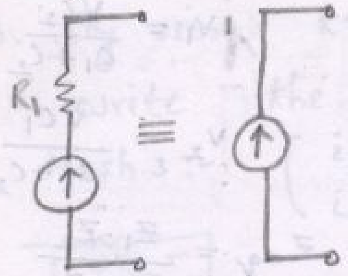
when $Z_A = Z_B = Z_C = Z$ then $Z_1 = Z_2 = Z_3 = 3Z$

ie Y – Δ transformation will increase the impedance by 3 times.

when $Z_1 = Z_2 = Z_3 = Z$, then $Z_A = Z_B = Z_C = \frac{Z}{3}$

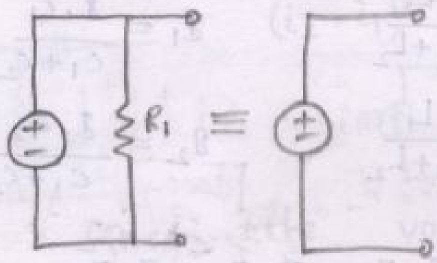
ie Δ – Y transformation will decrease the impedance by 3 times.

Equivalent circuits w.r.t. source point of view:-



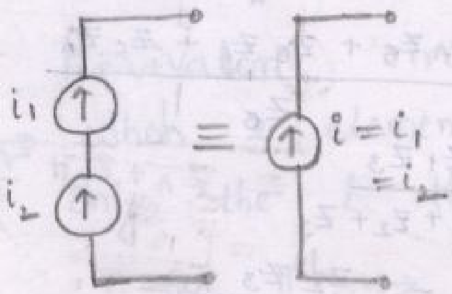
Here $R_1 \neq \infty$, since the violation of KCL.

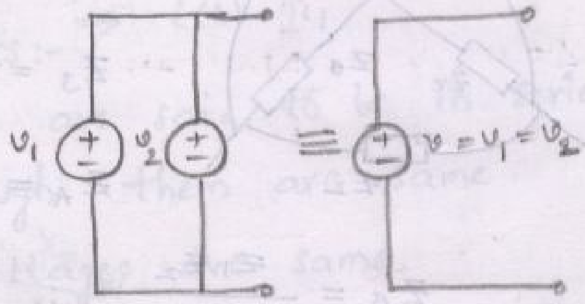
A resistor in series with an ideal CS, is neglected in the analysis. ie the load i ind. of R_1. We can't omit this R_1 in power calculations, since I^2R_1 is $\neq 0$.



Here $R_1 \neq 0$, since the violation of KVL.

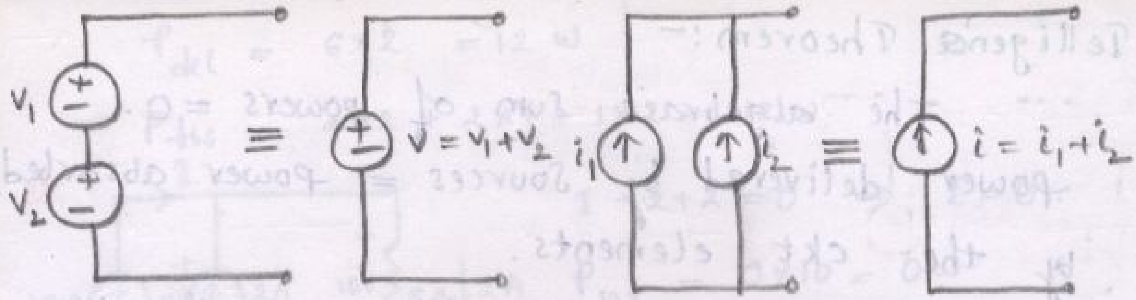
A resistor in $\parallel^{el}$ with an ideal VS can be neglected in the analysis ie the load volt. is ind. of R_1. We can't omit this R_1 in power calculations, since $V^2/R_1 \neq 0$.

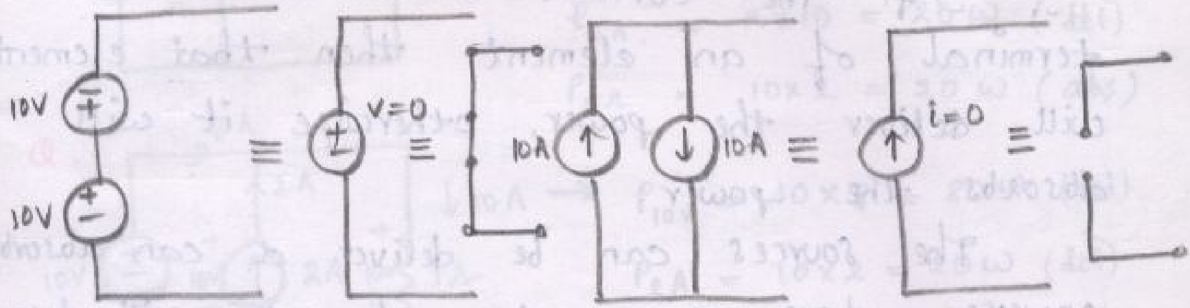




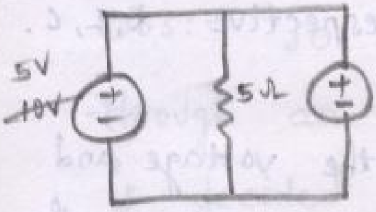
$-i_2 + i_1 = 0 \Rightarrow i_1 = i_2$ $\qquad$ $v_1 - v_2 = 0 \Rightarrow v_1 = v_2$

Two ideal CS are ~~able~~ ~~to~~ connected in series only when their magnitudes are equal, otherwise the violation of KCL, which results the unstability due to oscillations. Similarly 2 ideal VS are in $\parallel^{el}$ only when their magnitudes are equal, otherwise the violation of KVL.



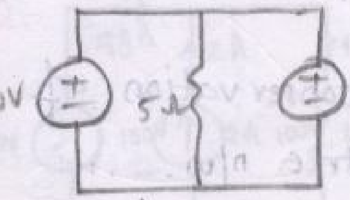


Q. Determine, the current through 5Ω, .

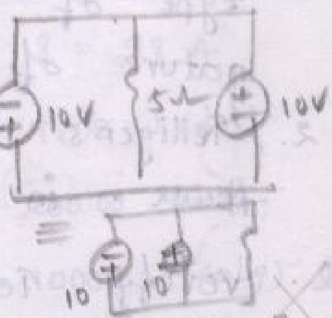


→ $i = \cancel{\frac{10}{5} = 2A}$ →

the violation in n/w and hence physical connection not possible.



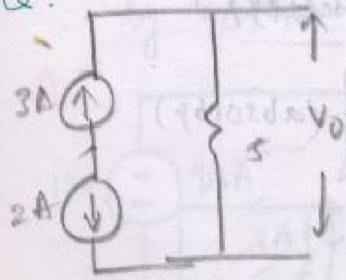
⇒ $i = \frac{10}{5} = 2A$



Not possible the physical connection.

By superposition theorem, n/w is not physically connected.

Q.



→ not possible

~~5×3~~

$2+3 = 0$ ✗

violation of KCL., n/w doesn't exists.

Telligen's Theorem:-

The algebraic sum of powers = 0.

power delivered by sources = power absorbed by the ckt elements.

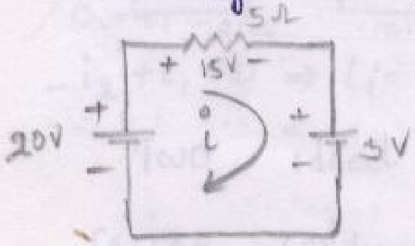
If the current enters at the -ve terminal of an element then that element will deliver the power, otherwise it will absorbs the power.

The sources can be deliver or can absorbs powers where as passive elements will always absorbs power., since the i will enter at the +ve terminal in the respective R, L, C.

Features:-

1. This theorem depends only on the voltage and current product in an element but not on the type of element [active or passive] ie ind. of nature of the element.
2. Telligen's th. expresses the conservation of power in every lumped electric n/w.

Q. verify Telligen's Th.



$$20 - 5i - 5 = 0$$

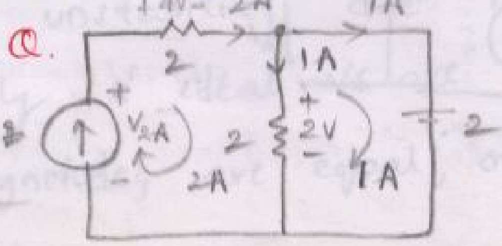
$$\Rightarrow i = \frac{20-5}{5} = 3A$$

$$P_{20V} = 20 \times 3 = 60 \text{ (delivers)}$$

$$\left.\begin{array}{l} P_R = 15 \times 3 = 45 \\ P_{5V} = 5 \times 3 = 15 \end{array}\right\} \text{(absorbs)}$$

$\therefore$ $P_{del} = P_{abs}$.

Q.



Sol:

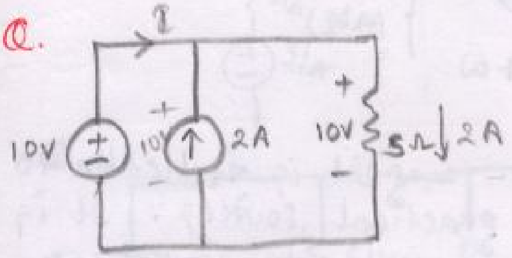
$V_{2A} - 4 - 2 = 0$

$\Rightarrow V_{2A} = 6V$

$P_{del} = 6 \times 2 = 12\ W$

$P_{abs} = 4 \times 2 + 2 \times 1 + 1 \times 2 = 12\ W$

Q.



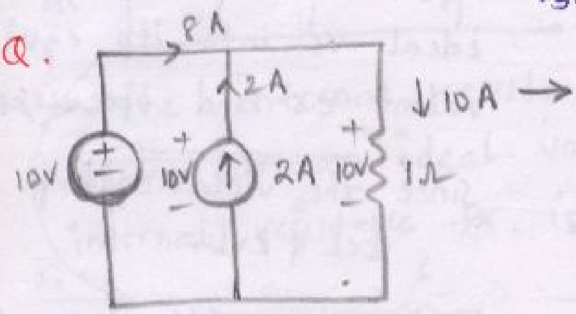
$I - 2 + 2 = 0 \Rightarrow I = 0.$

$P_{10V} = 0 \times 10 = 0\ W$

$P_{2A} = 2 \times 10 = 20\ W$ (del)

$P_{5\Omega} = 10 \times 2 = 20\ W$ (abs)

Q.



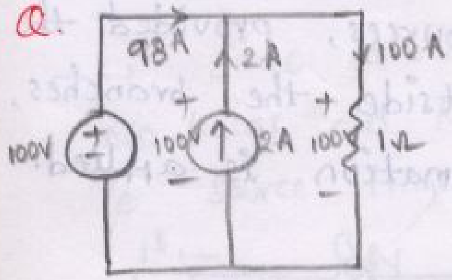
$\rightarrow P_{10V} = 10 \times 8 = 80\ W$ (del)

$P_{2A} = 10 \times 2 = 20\ W$ (del)

$P_{1\Omega} = 10 \times 10 = 100\ W$ (Abs)

Obs:- so from the above 2 problems, the i through an ideal VS can be any value, it is decided by the other elements magnitudes present in the n/w.

Q.



$-I - 2 + 100 = 0 \Rightarrow I = 98\ A$

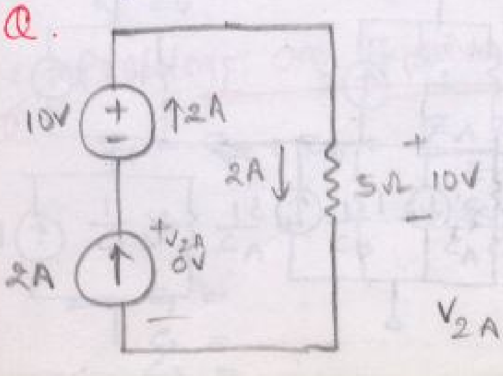
$P_{100V} = 100 \times 98 = 9800\ W$ (del)

$P_{2A} = 100 \times 2 = 200\ W$ (del)

$P_{1\Omega} = 100 \times 100 = 10000\ W$ (abs)

→ from the above problems, the voltage across CS can be any value, it is decided by other element present in the n/w.

Q.



$P_{2A} = 2 \times 0 = 0\ W$

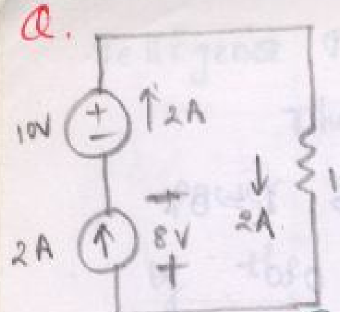
$P_{10V} = 10 \times 2 = 20\ W$ (del)

$P_{5\Omega} = 2 \times 10 = 20\ W$ (abs)

$V_{2A} + 10 - 10 = 0$

$\Rightarrow V_{2A} = 0\ V$

Q.



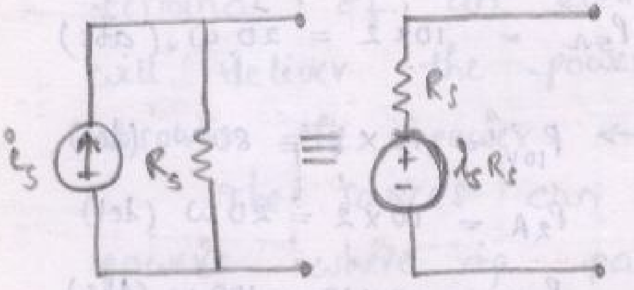
$\longrightarrow P_{10V} = 20\,\omega$ (del)

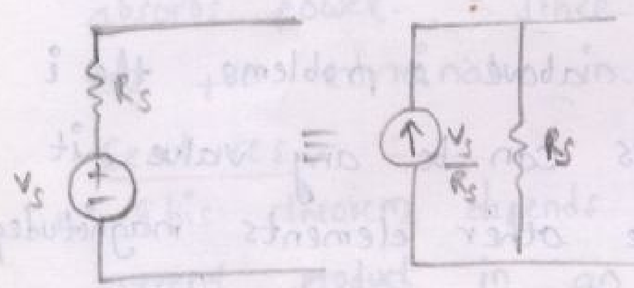
$P_{2A} = 16\,\omega$ (ab)

$P_{1\Omega} = 4\,\omega$

Source Transformation :-

→ It is applicable to practical sources. It is impossible to convert an ideal vs into its equivalent cs and viceversa, since the violations of kcl & kvl.

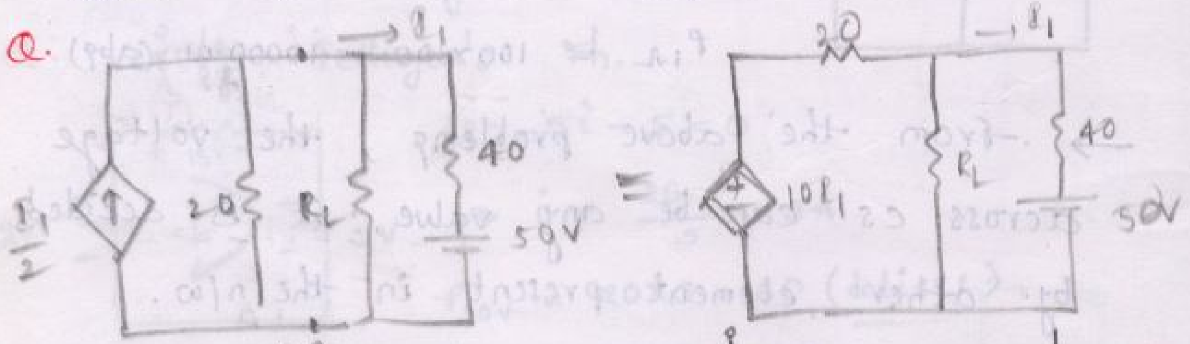




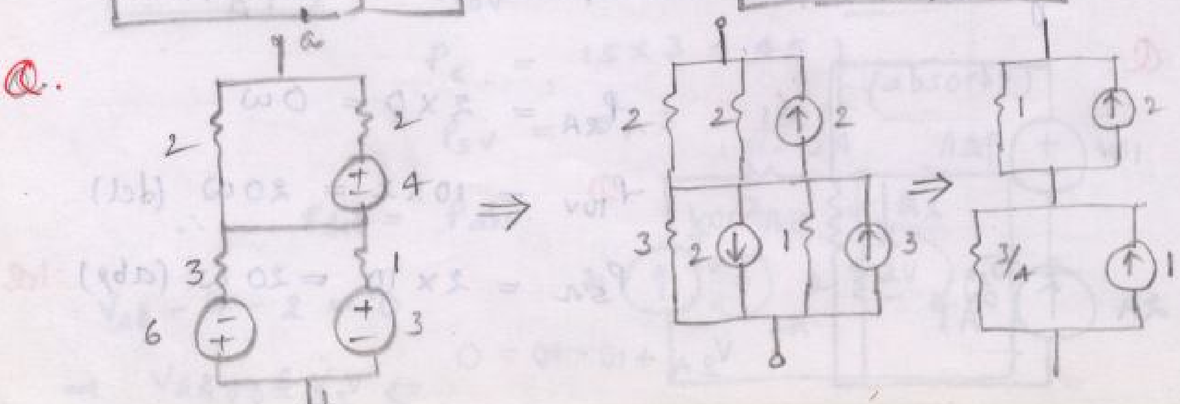
The above ckts are equal only w.r.t. the performance point of view. But the elements in the connection point of view they are not equal.

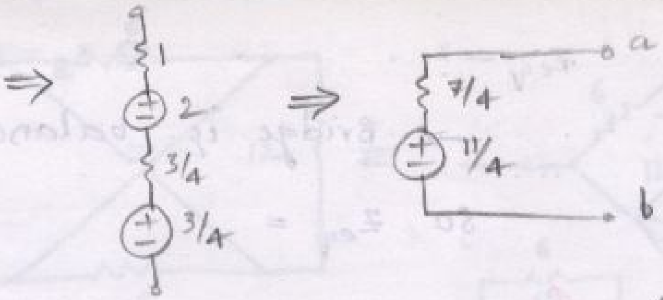
→ The source transformation is applicable even for the dependent sources, provided the controlled variable is outside the branches, where the source transformation is applied.

Q.

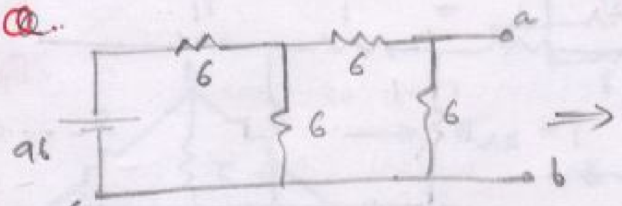


Q.





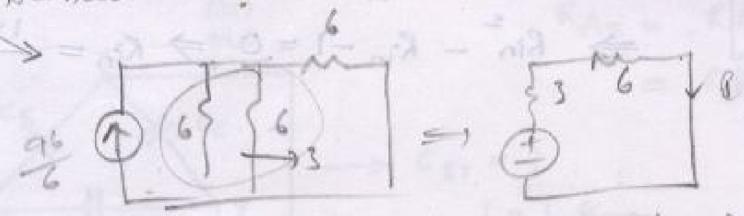
Q.



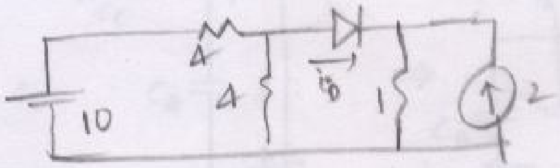
An ideal ammeter is connected across ab, then the reading of the ammeter is ?

→ The internal resistance of an ideal ammeter is zero for an ideal voltmeter is infinite.

internal resistance for ideal cs → ∞, for ideal vs → 0

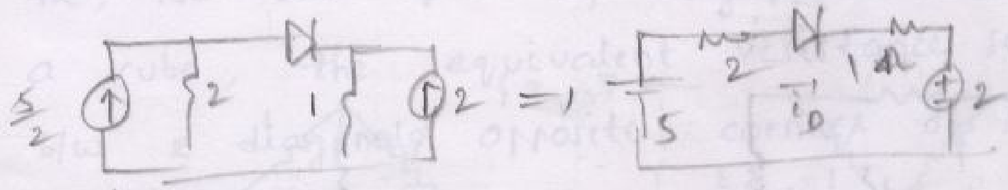


Q.



→ i_D = ?

→ Since diode is a non-linear element, n/w is non linear and hence superposition is not applicable. ie source transformation is applicable.



ideal D

$V_f = 0$

$R_f = 0$

$5 - 2i - 1i - 2 = 0$

$3 - 3i = 0$

$\Rightarrow i = 1A$

Problems on Equivalent circuits

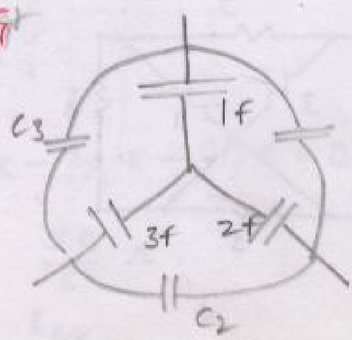
Q. $Z_1 = Z_A + Z_B + \dfrac{Z_A Z_B}{Z_C}$

$\dfrac{1}{C_1} = \dfrac{1}{C_A} + \dfrac{1}{C_B} + \dfrac{C_C}{C_A C_B}$

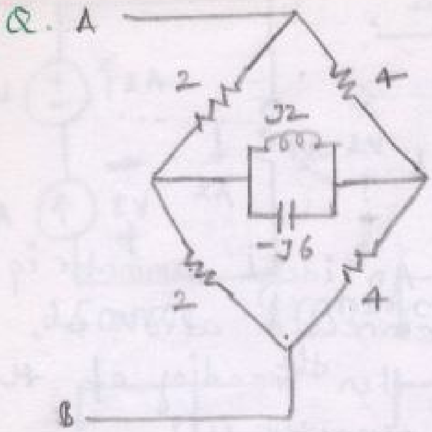
$\Rightarrow C_1 =$

$C_2 =$

$C_3 =$



Q.



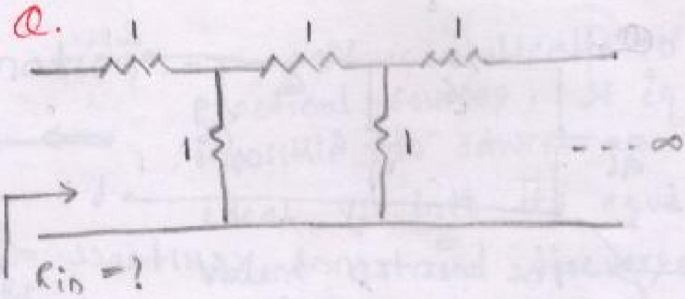
Z_{eq} — ?

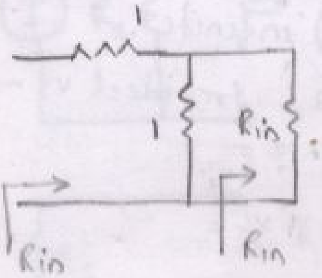
$Z_1 Z_3 = Z_2 Z_4$

→ Bridge is balanced,

so Z_{eq} =

Q.

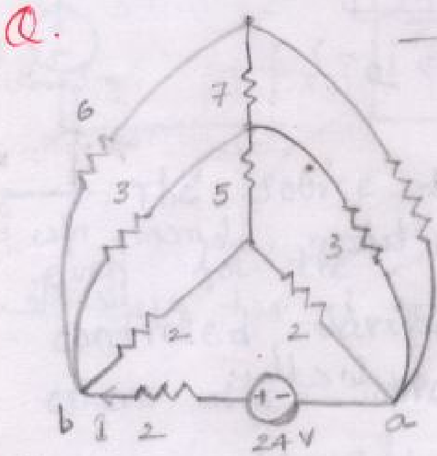




$$R_{in} = 1 + \frac{R_{in}}{1+R_{in}}$$

$$\Rightarrow R_{in}^2 - R_{in} - 1 = 0 \Rightarrow R_{in} = \frac{1+\sqrt{5}}{2}$$

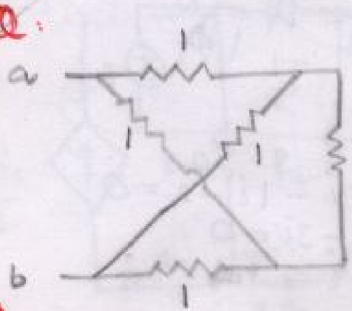
Q.



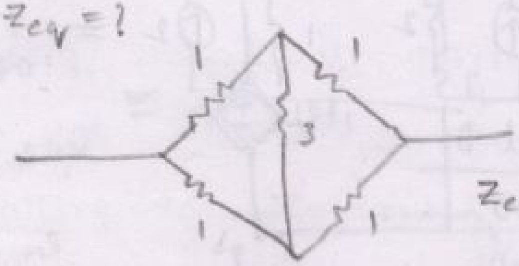
→ $I = ?$

$$\rightarrow I = \frac{24}{2 + 4 \| 6 \| 12} = 6A$$

Q.

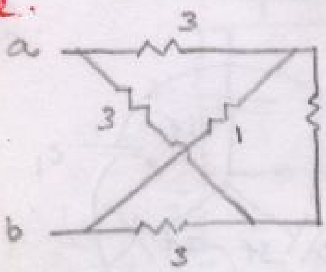


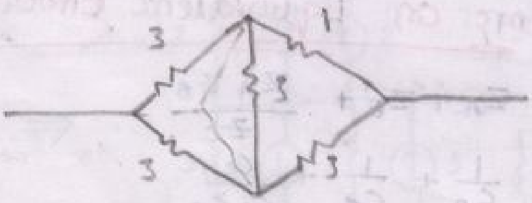
→ $Z_{eq} = ?$



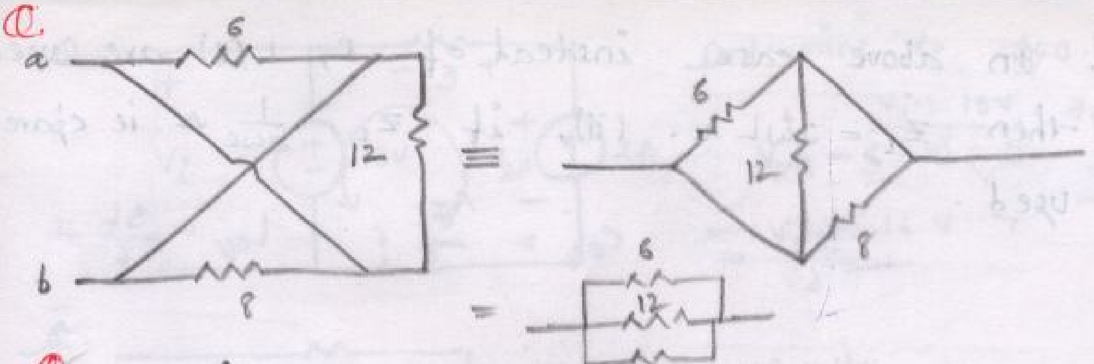
$Z_{eq} = 1\,\Omega$

Q.

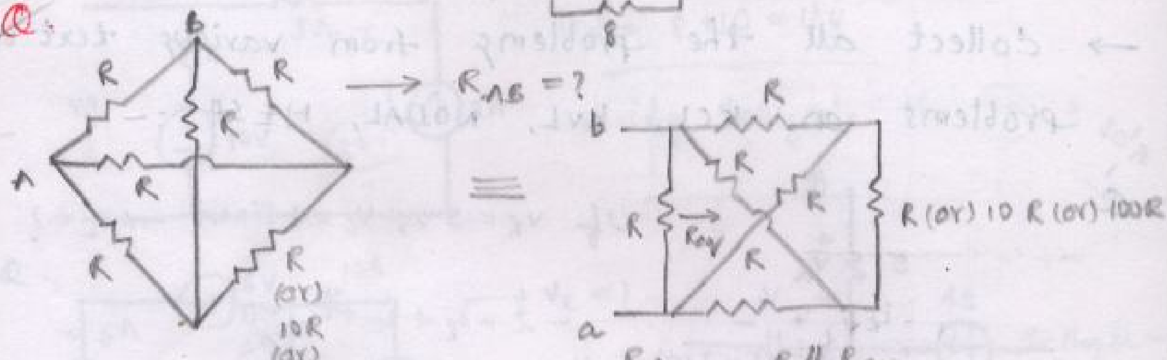




Q.



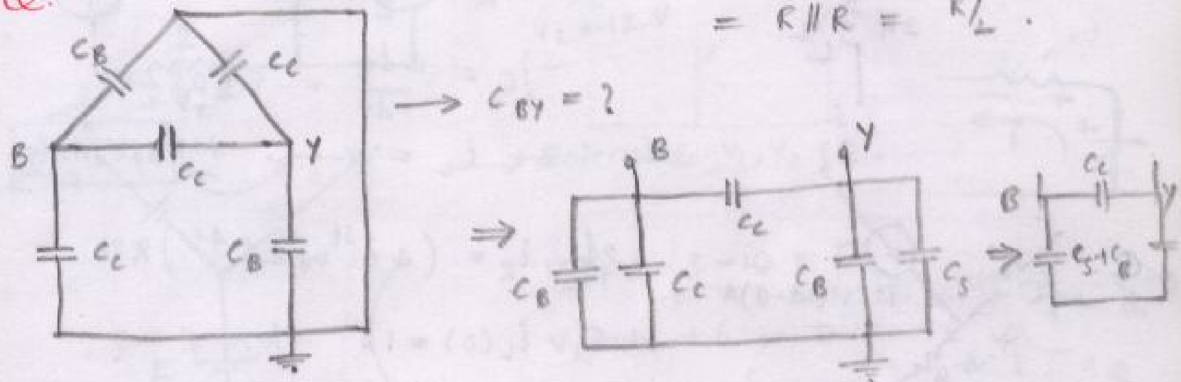
Q.



$R_{AB} = R \parallel R_{eq}$

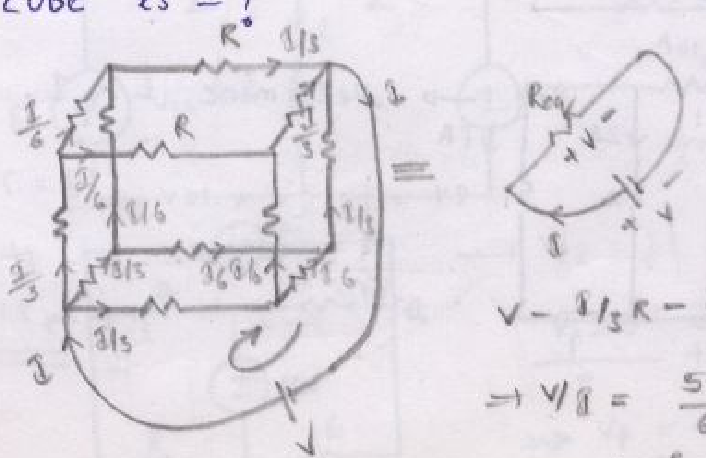
$= R \parallel R = R/2$.

Q.



$$\Rightarrow C_{BY} = \frac{C_B + C_C}{2} + C_C$$

Q. 12, 1Ω are used as edges to form a cube, the equivalent resistance seen b/w 2 diagonals opposite corners of the cube is — ?



$R_{eq} = \frac{V}{I}$

$V - I/3\, R - I/6\, R - I/3\, R = 0$

$\Rightarrow V/I = \frac{5R}{6}\ \Omega$.

$= R_{eq}$

Q. In above case instead of R, L(H) are used. then $Z_L = j\omega L\ \Omega$. (ii). if $Z_L = \frac{1}{j\omega C}\ \Omega$ ie C's are used.

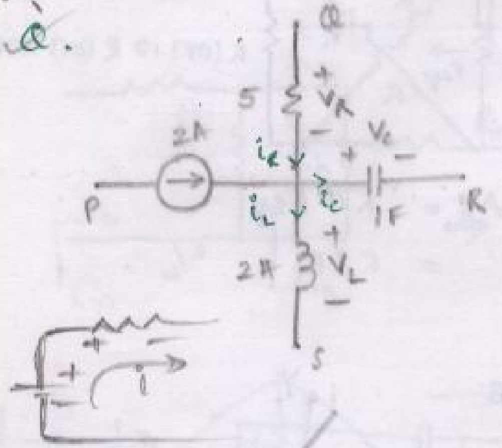
$\frac{5Z_c}{6}$ = $C_{eq} = \frac{6C}{5}$ f $L_{eq} = \frac{5L}{6}$ H

→ collect all the problems from various text books.

problems on KCL, KVL, NODAL, MESH :-

G'03

Q.



If $V_R = 5$ V, $V_c = 4\sin\omega t$ → $V_L = ?$

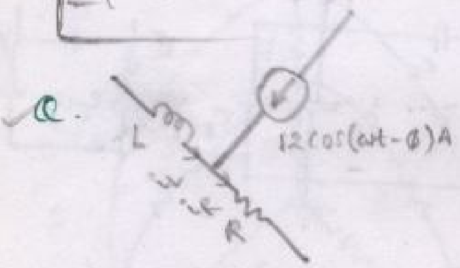
$-2 - i_R + i_c + i_L = 0$

$i_R = \frac{V_R}{R} = 1$ A

$i_C = C\frac{dv_c}{dt} = 8\cos 2t$

→ $i_L = $ ——. Ans $\boxed{32\sin 2t}$

Q.



If $i_R = (4e^{-3t} + 3e^{-4t})$ A

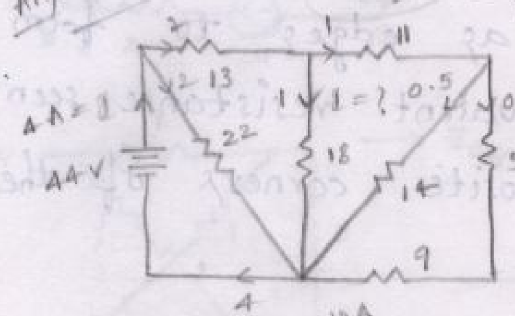
& $i_L(0) = 1$ A then $\phi = ?$

$-i_L - 12\cos(\omega t - \phi) + i_R = 0$

At $t=0$ ⟹ $-i_L(0) - 12\cos(-\phi) + i_R(0) = 0$

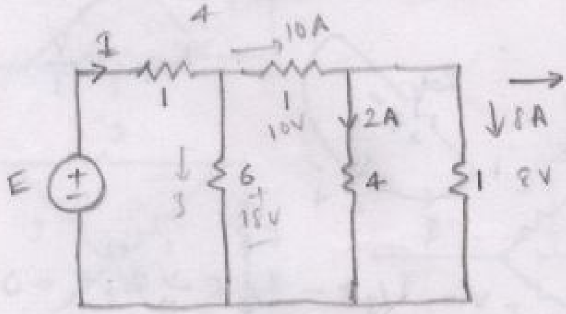
Ans $\phi = 60°$

Q.



→ $i_{18\Omega} = ?$

Q.



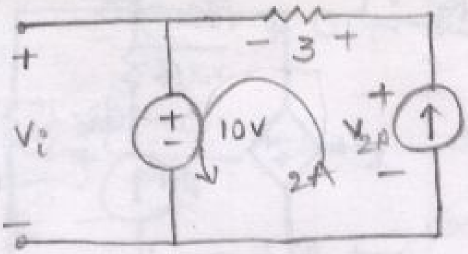
→ Determine E & I.

8V + 10V = 18V

I = 3 A

E = 18 + 3 V = 21 V

Q.



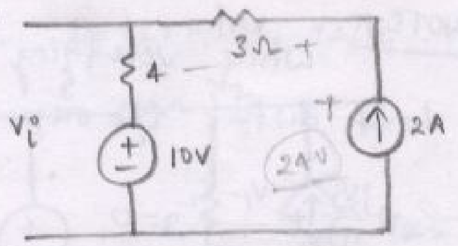
Determine V_P also verify Tellegen's theorem.

$V_i = 10V$

$V_{2A} - 6 - 10 = 0$

$\Rightarrow V_{2A} = 16 V$

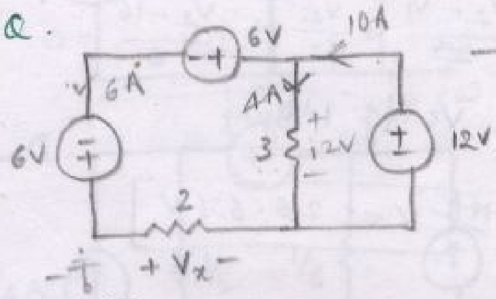
Q.



$V_i = 8 + 10 = 18V$

verify Tellegen's th.

Q.

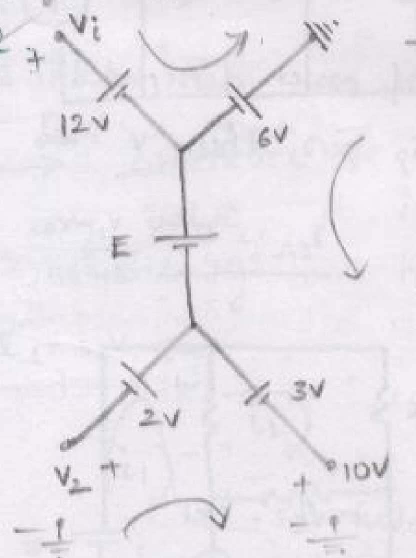


$\longrightarrow V_x = ?$

$V_x - 6 + 6 - 12 = 0$ verify Tellegen

$V_x = 12 V$

Q



$\longrightarrow$ Determine V_1, V_2 & E.

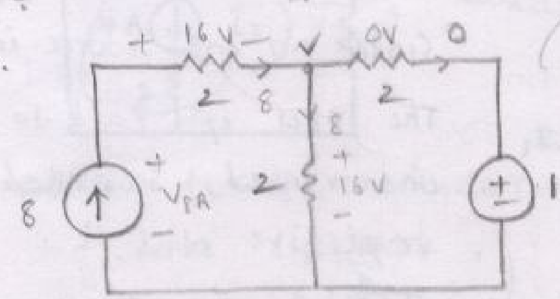
$V_2 + 2 - 3 - 10 = 0$

$V_1 - 12 + 6 = 0$

$-10 - 6 - E - 3 = 0$

$V_P - V_{PQ} - V_Q = 0$

Q.



$V_{8A} - 16 - 16 = 0$

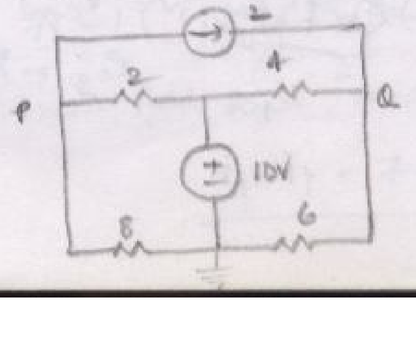
$-8 + \frac{V}{2} + \frac{V-16}{2} = 0$

$V = 16$

$I = V/_2 = 8A$

G'03

Q.



$\longrightarrow V_{PQ} = ?$

$\frac{V_Q - 0}{6} + \frac{V_Q - 10}{4} - 2 = 0$

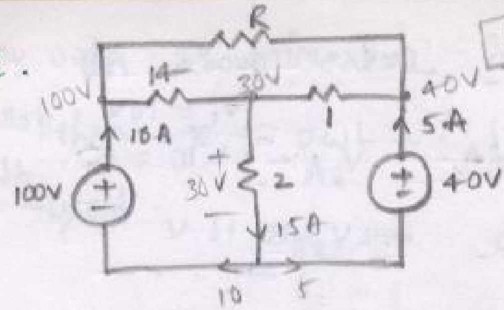
$\Rightarrow V_Q = 10.8V$

$\frac{V_P - 0}{8} + \frac{V_P - 10}{2} + 2 = 0$

$\rightarrow V_P = 4.8V$

$V_{PQ} = V_P - V_Q = -6V$

G'03 Q.

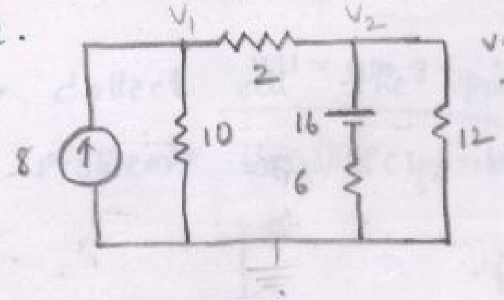


$\boxed{R = ?}$

$$-10 + \frac{100-40}{R} + \frac{100-30}{14} = 0$$

$\rightarrow R = \_\_$.

Q.

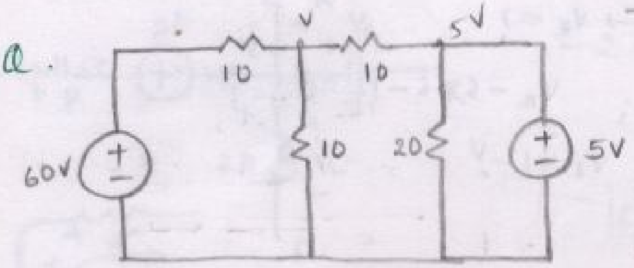


verify Telligen NOTE: while verifying Telligen don't disturb the original n/w

$$-8 + \frac{V_1}{10} + \frac{V_1 - V_2}{2} = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{12} + \frac{V_2 - 16}{6} = 0$$

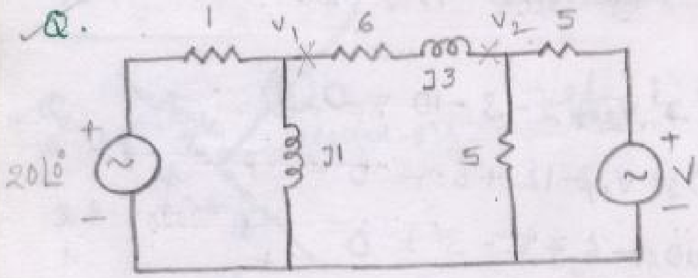
Q.



verify Telligen

$\boxed{V = 26.67\,V}$

Q.



If power dissipated in 6Ω is zero then 'V' = ?

$I_{6\Omega} = 0$, $\frac{V_1 - V_2}{6 + J3} = 0$

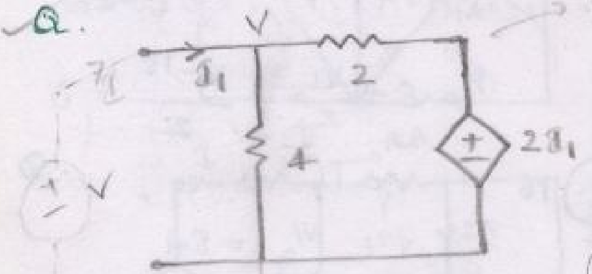
$\Rightarrow V_1 = V_2$

$$\frac{V_1 - 20\angle 0^\circ}{1} + \frac{V_1}{J1} = 0 \rightarrow V_1 =$$

$$\frac{V_2 - V}{5} + \frac{V_2}{5} = 0 \xrightarrow{\text{sub } V_2 = V_1}$$

Q.

what about the ckt



Since no ind. source in n/w, The n/w is said to be unenergized, so called resistive n/w.

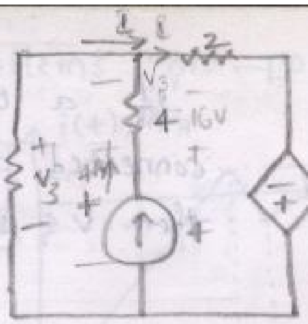
$V = R_{eq} I$

$R_{eq} = V/I = ?$

$I = I_1$

$$-I_1 + \frac{V}{4} + \frac{V - 2I_1}{2} = 0$$

$\Rightarrow V/I = \_\_$.

Q. 

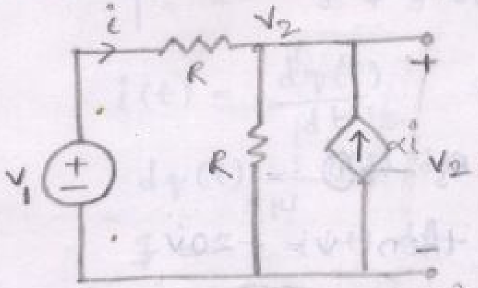
Determine I.

$$\frac{V_3}{3} - 4 + \frac{V_3 + 4V_3}{2} = 0$$

$$\Rightarrow V_3 =$$

$$I = \frac{V_3 + 4V_3}{2} = \frac{60}{17}\ A$$

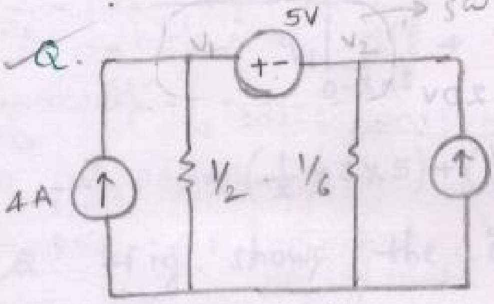
Q. Determine V_2/V_1.



$$\frac{V_2 - V_1}{R} + \frac{V_2}{R} - \alpha i = 0$$

where $i = \dfrac{V_1 - V_2}{R}$

$$V_2/V_1 = \frac{1+\alpha}{2+\alpha}$$

Q. 

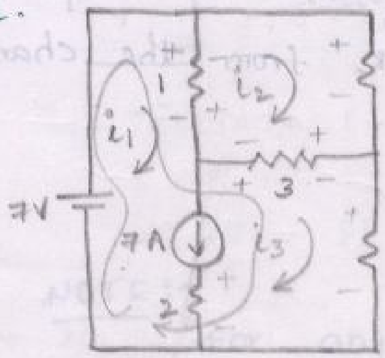
→ Determine V_1 & V_2.

$(V_1 - V_2 = 5) \leftarrow V_1 - 5 - V_2 = 0$

$$-4 + \frac{V_1}{1/2} + \frac{V_1}{1/6} - 9 = 0$$

In side Super node always KVL is written.

→ ~~Whenever the~~ Since i through an ideal VS can be any value, it is not possible nodal eq at ①, ② nodes independently and hence super node procedure is followed.

Q. 

→ Determine power dissipated in 3Ω resistor.

~~Also use~~

$$-2i_2 - 3(i_2 - i_3) - 1(i_2 - i_1) = 0$$

Since the volt across any ideal CS can be any value it is not possible to write mesh eq for the meshes ① & ③ indly and hence supermesh.

$$7 - 1(i_1 - i_2) - 3(i_3 - i_2) - 1(i_3) = 0$$

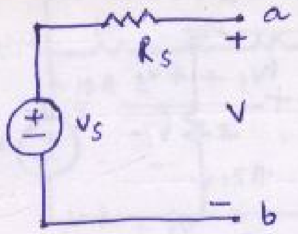
→ In side super mesh KCL

$$i_1 - i_3 = 7\ A$$

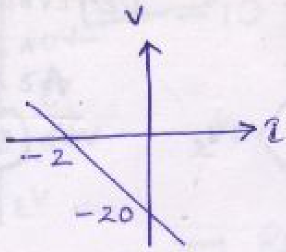
$$P_{3\Omega} = (i_2 - i_3)^2\, 3$$

$$P_{3\Omega} = 0.75\ W$$

Q.



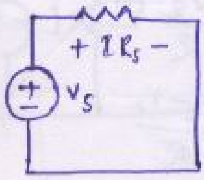
fig(a)



fig(b).

If a 10Ω is connected across a,b then V & I — ?

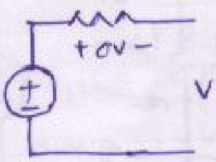
(−2, 0) ⇒ when voltage V = 0, then I = −2A



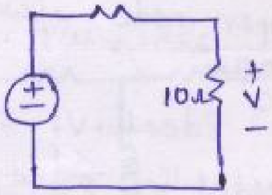
$V_s - IR_s = 0$

$\Rightarrow V_s = -2R_s \longrightarrow ①$

(0, −20) ⇒ when I = 0, then V = −20V



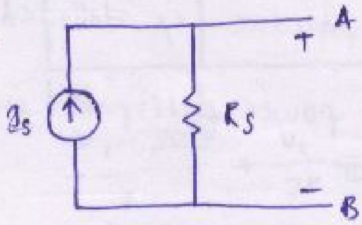
$V = V_s$ $\therefore V_s = -20V$



$V = 10I$

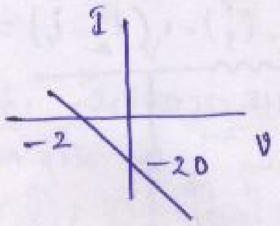
$I = \frac{V_s}{10+10} = -1A$

Q.



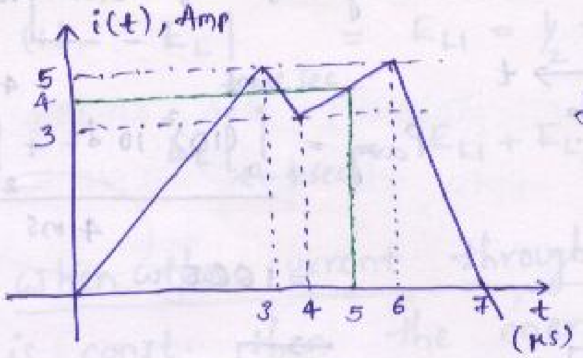
If a 10Ω resistor is connected across the a,b terminals then V & I— find I_s, R_s from the char-s.

fig(a)



Problems on power and energy:-

Q. fig. shows the i flowing through the capacitor. Determine the charge acquired by capacitor upto the first $5\,\mu sec$ — ?



$$i(t) = \frac{dq(t)}{dt}$$

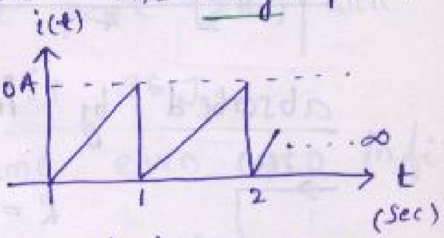
$$dq(t) = i(t).dt$$

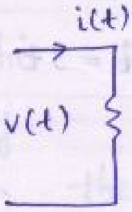
$$q = \int_0^{5\mu s} i(t).dt = \text{Area under } i(t) \text{ upto } 5\mu Sec$$

$$= q_1\Big|_{0-3\mu s} + q_2\Big|_{3-4\mu s} + q_3\Big|_{4-5\mu s}$$

$$= \left(\tfrac{1}{2}\times 3\times 5\right) + \left(\tfrac{1}{2}\times 1\times 2 + 1\times 3\right) + \left(\tfrac{1}{2}\times 1\times 1 + 1\times 3\right) = 15\,\mu C.$$

Q. fig. shows the i through 10Ω. The avg. power dissipated by resistor is—?





P_{avg} = (Energy absorbed over one period)/(period)

$$= \frac{\int_0^1 i^2 R\, dt}{1\ sec} = \frac{\int_0^1 (10t)^2 .10.dt}{1}$$

$$= \frac{1000}{3}\ J/sec$$

NOTE:- for any general period 'T', (1). for a voltage wave form $P_{avg} = \dfrac{\int_0^T \frac{v^2}{R}\,dt}{T} = \dfrac{\frac{1}{T}\int_0^T v^2 dt}{R}$

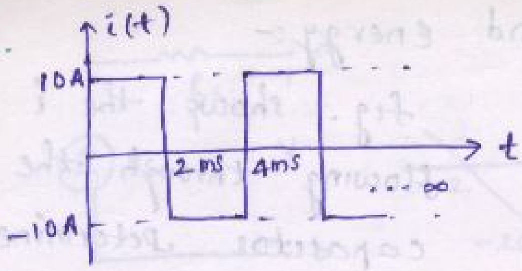
$$\Rightarrow P_{avg} = \frac{V_{rms}^2}{R}\ (\omega)$$

(2). for a current wave form $P_{avg} = \dfrac{\int_0^T i^2 R\, dt}{T}$

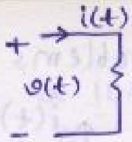
$$= \tfrac{1}{T}\int_0^T i^2 dt \;.\, R$$

$$\therefore P_{avg} = I_{rms}^2 \cdot R\ \ (\omega).$$

Q.



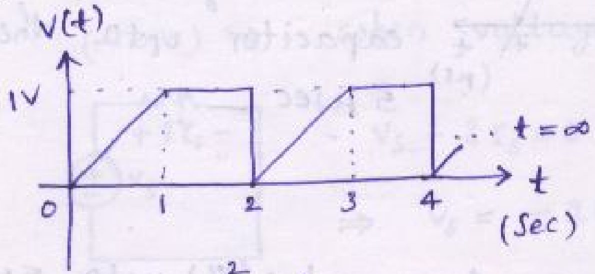
$P_{avg} = ?$ if

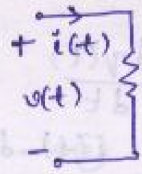


$$P_{avg} = \frac{\int_0^{2ms} (10)^2 \cdot 10\, dt + \int_{2ms}^{4ms} (-10)^2 \cdot 10\, dt}{4\ ms}$$

$= 1000\ W$

Q.





$$P_{avg} = \frac{\int_0^2 \frac{v^2}{R} dt}{2\ sec} = \frac{1}{2}\left[\int_0^1 \frac{t^2}{10} \cdot dt + \int_1^2 \frac{1}{10} dt\right] = \frac{1}{15}\ W$$

G'03

Q.

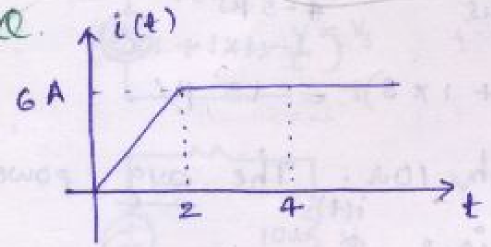
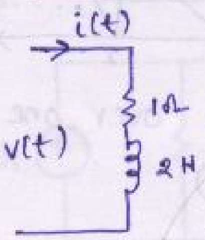


fig. shows the i flowing through an inductor of 2H and resistor of 1Ω. Determine ~~avg. power dissipated~~ energy absorbed by inductor upto the first 4 sec – ?



$R = 1\Omega$, $0 \le t \le 2\ sec$, $i = 3t$

$$E_{R_1} = \int_0^2 i^2 R\, dt \cdot J$$

$$= \int_0^2 (3t)^2 \cdot 1 \cdot dt = 24\ J$$

This is not a periodic one

$$E_{R_2} = \int_2^4 6^2 \cdot 1 \cdot dt = 72\ J$$

$L = 2H$, $0 \le t \le 2\ sec$, $i = 3t$ $\quad v = L\frac{di}{dt}$

$$E_{L_1} = \int_0^2 \underbrace{Li\frac{di}{dt}}_{v \cdot i} \cdot dt \cdot (J) = \int_0^2 2 \cdot 3t \cdot 3 \cdot dt = 36\ J$$

$$E_{L_2} = \int_2^4 2 \cdot 6 \cdot (0) \cdot dt = 0\ J. \quad \left(2 \text{ to } 4,\ i = 6,\ \frac{di}{dt} = 0\right)$$

$$E_{abs}\Big|_{4\ Sec} = E_{R_1} + E_{R_2} + E_{L_1} + E_{L_2}$$

$= 132\ J$

NOTE :-

(1) .. $E_L\Big|_{t=2\,sec} = E_{L1} = \frac{1}{2}\times 2\times 6^2 = 36\,J$

$E_L\Big|_{t=4\,sec} = E_{L1} + E_{L2} = \frac{1}{2}$

when the current through an ideal inductor is const. then the energy absorbed zero, since the instantaneous power $P = Li\frac{di}{dt} = 0$

similarly for a const. capacitive voltage the energy absorbed is zero, since instantaneous power $\Rightarrow P = Cv\cdot\frac{dv}{dt} = 0$.

Q. In the above problem the energy stored by inductor (1Ω, 2H) upto the first 4 sec — ?

Only the ideal inductive part [2H] will store the energy, so it is 36 J.

This stored energy is same even upto infinity.

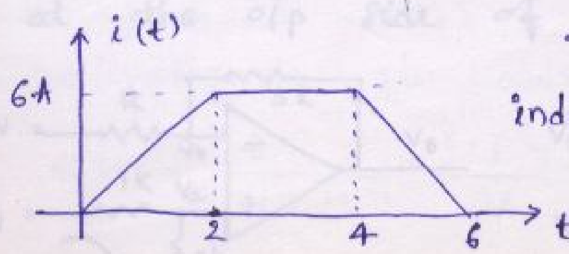
Q. In the above case the energy absorbed by the inductor (1Ω, 2H) upto infinity is — ?

$$E_{abs}\Big|_{t=\infty} = E_R\Big|_{t=\infty} + E_L\Big|_{t=\infty}$$

$$= 24 + \int_2^{\infty} 6^2\cdot 1.dt + 36 + 0$$

$$= 60 + 36(\infty - 2) = \infty.$$

Q.



The energy stored by the inductor upto the first 6 sec — ?

$E_L\Big|_{t=6} = \frac{1}{2}\times 2\times(0)^2 = 0\,J$

(or) $E_L\Big|_{\substack{stored\\t=6}} = E_{L_1} + E_{L_2} + E_{L_3} = 36 + 0 - 36$
$= 0\ J.$

Q. In the above problem, the energy absorbed by the inductor upto the first 6 sec — ?

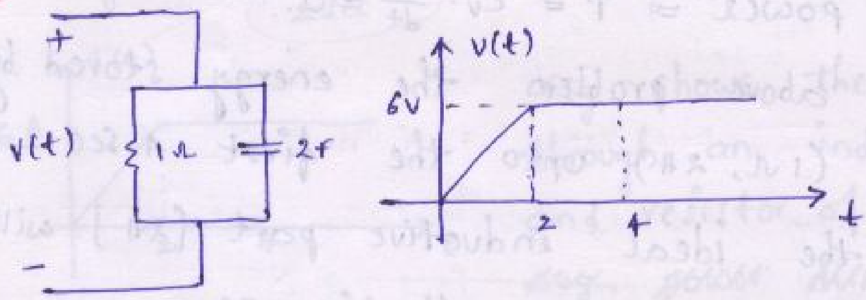
$$E_{abs}\Big|_{t=6} = E_{R_1} + E_{R_2} + E_{R_3} + E_{L_1} + E_{L_2} + E_{L_3}$$

$$= 24 + 72 + 24 + 36 + 0 - 36$$

$$= 120\ J.$$

$$E_{R_3} = \int_4^6 i^2 R\, dt = \int_4^6 [-3(t-6)]^2 \cdot 1 \cdot dt = 24\ J.$$

Q.



$$E_R = \int \frac{v^2}{R}\, dt \cdot (J) \qquad E_c = \int cv\left(\frac{dv}{dt}\right) dt \cdot (J)$$

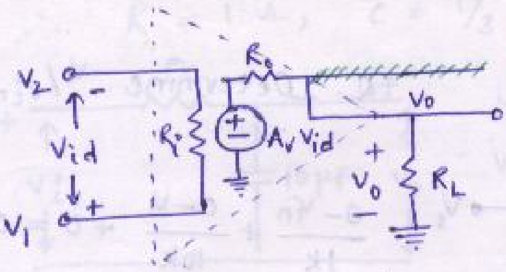
(i). $E_{abs}\Big|_{t=4} = 132\ J$ (ii). $E_{stored}\Big|_{\substack{t=4\\ (or)\ t=\infty}} = 36\ J$

(iii). $E_{abs}\Big|_{t=\infty} = \infty\ J$

(iv). $E_{sto}\Big|_{t=6sec} = 0\ J$

(v). $E_{abs}\Big|_{t=6sec} = 120\ J.$

Op-Amp's [Ideal] :-

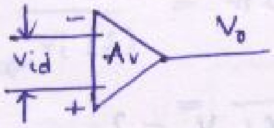


By writing KVL at the i/p side,

$V_1 - V_{id} - V_2 = 0$

$\Rightarrow V_{id} = V_1 - V_2$

- for any open loop voltage amplifier



$V_o = A \cdot V_{id} \Rightarrow A_v = \frac{V_o}{V_{id}}$

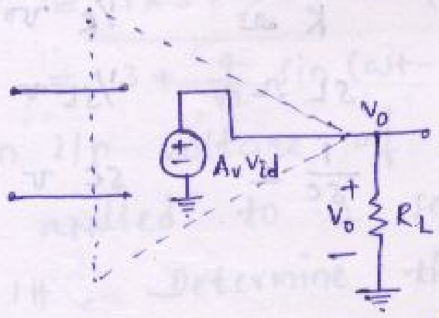
$\Rightarrow V_{id} = \frac{V_o}{A_v}$

- for an ideal op-amp, $A_v = \infty$

$\Rightarrow V_{id} = 0 \Rightarrow V_1 - V_2 = 0 \Rightarrow V_1 = V_2$.

So the voltage at non-inverting terminal = volt. at inverting terminal.

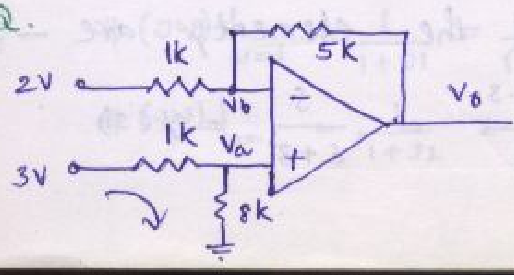
- for an ideal op-amp $R_i = \infty$ & $R_o = 0$.



So the i/p current accepted by an ideal opamp = 0. And hence it is possible to write the nodal eq^n at the i/p side of an ideal op-amp.

Since $R_o = 0$, there exists an ideal VS at the o/p side of an ideal op-amp and the current through it can be any value, it is not possible to write the nodal eq^n at the o/p side of an ideal op-amp.

Q.



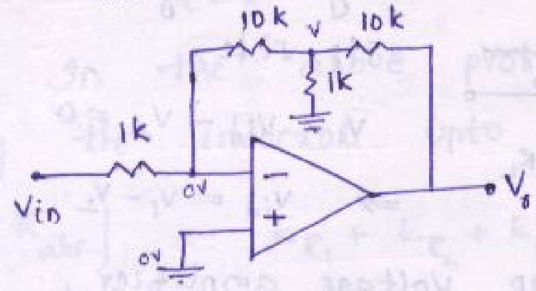
$V_a = \frac{3}{1k + 8k} \cdot 8k = \frac{8}{3} V$

$V_b = V_a = 8/3 \ V$

$$\frac{8/3 - 2}{1\times10^3} + \frac{8/3 - V_0}{5\times10^3} = 0 \Rightarrow V_0 =$$

G'04

Q.



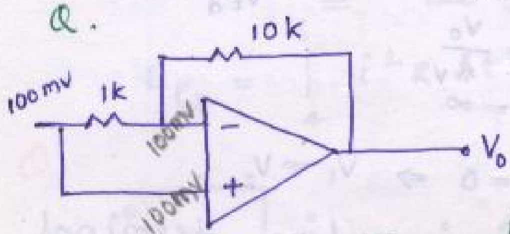
Q. Determine V_0/V_{in}

$$\frac{0 - V_{in}}{1k} + \frac{0 - V}{10k} + 0 = 0$$

$$\Rightarrow V = -10\,V_{in}$$

$$\frac{V - 0}{10k} + \frac{V}{1k} + \frac{V - V_0}{10k} = 0 \Rightarrow$$

Q.



Determine V_0 — ?

Ans: 100 mV.

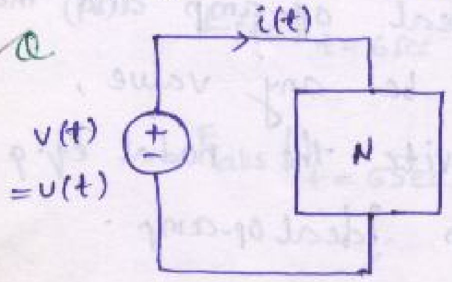
Laplace Transforms:-

1. $v(t) \xrightarrow{L.T} V(s)$ $\quad Z(s) = \frac{V(s)}{I(s)} \quad Y(s) = \frac{I(s)}{V(s)}$
2. $i(t) \longrightarrow I(s)$
3. $\delta(t) \longrightarrow 1$
4. $u(t) \longrightarrow 1/s$
5. $R \longrightarrow R$
6. $L \longrightarrow sL$
7. $C \longrightarrow \frac{1}{sC}$

$Z(s)$	$Y(s)$
$R\ \Omega$	$1/R\ \mho$
$sL\ \Omega$	$1/sL\ \mho$
$\frac{1}{sC}\ \Omega$	$sC\ \mho$

The impedances in series and admittances in $\|^{el}$ we can add.

Q.



The n/w N contains only 2 elements. The response for the unit step excitation is $i(t) = e^{-3t}$ Amp for $t \geq 0$. Then the elements are — ?

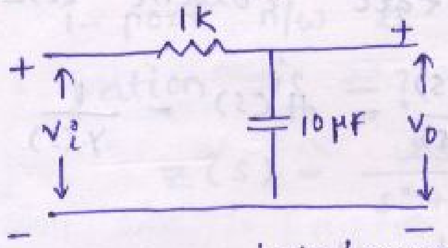
$$\frac{I(s)}{V(s)} = H(s) = \frac{1/(s+3)}{1/s} = \frac{s}{s+3} = Y(s)$$

$$Z(s) = \frac{s+3}{3} = 1 + \frac{3}{s} = 1 + \frac{1}{s/3} = R + \frac{1}{sC}$$

$\therefore$ $R = 1\,\Omega$, $C = 1/3\,F$ are in series.

G'04 EC

Q. Determine the s.s. o/p voltage, corr. to the i/p $3 + 4\sin 100t$ is.



$$\frac{V_0(s)}{V_i(s)} = H(s) \Rightarrow V_o = \frac{V_i}{R + \frac{1}{sC}} \cdot \frac{1}{sC}$$

($V_0(s)$ → desired response; $V_i(s)$ → excitation)

$$= \frac{1}{1 + sCR}$$

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

$$H(j\omega)\Big|_{\omega=0} = \frac{1}{1+0} = 1$$

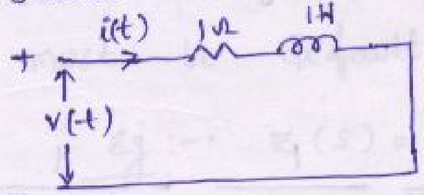
$$H(j\omega)\Big|_{\omega=100} = \frac{1}{1+j1} = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$\therefore V_o = 1 \times 3 + \frac{1}{\sqrt{2}} \cdot 4 \sin(\omega t - \pi/4)$$

$$= 3 + \frac{4}{\sqrt{2}} \sin(\omega t - \pi/4)\ V.$$

G'04 EC

Q. An i/p voltage of $v(t) = 10\sqrt{2}\cos(t + 10^\circ) + 10\sqrt{5}\cos(2t + 10)\,V$ is applied to a series combi. of $R = 1\,\Omega$ & $L = 1H$. Determine the resulting steady state i.



$$\frac{I(s)}{V(s)} = H(s) = \frac{1}{R + sL}$$

$$H(s) = \frac{1}{s+1}$$

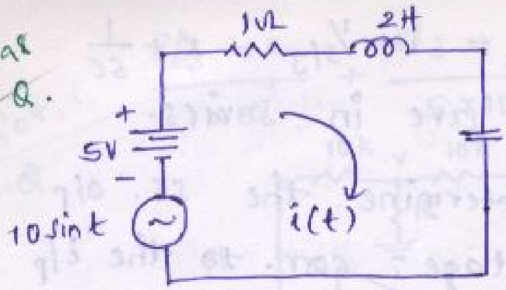
$$H(j\omega) = \frac{1}{1 + j\omega}$$

$$H(j\omega)\Big|_{\omega=0} = 1$$

$$H(j\omega)\Big|_{\omega=1} = \frac{1}{1+j1} = \frac{1}{\sqrt{2}} \angle -45^\circ$$

$$H(j\omega)\Big|_{\omega=2} = \frac{1}{1+j2} = \frac{1}{\sqrt{5}} \angle -63.43$$

G '98 Q.



→ Determine the steady state current i(t) − ?

$$\frac{I(s)}{V(s)} = H(s) = \frac{1}{Y(s)}$$

$$\therefore \quad H(s) = \frac{1}{R + sL + \frac{1}{sC}}$$

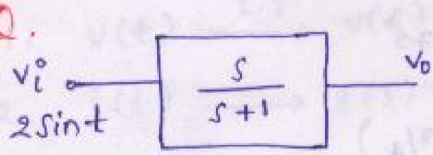
$$H(j\omega) = \frac{1}{R + j\omega L + \frac{1}{j\omega C}}$$

$$H(j\omega)\Big|_{\omega=0} = \frac{1}{1 + j0 + \frac{1}{j0}} = 0$$

$$H(j\omega)\Big|_{\omega=1} = \frac{1}{1 + j2 + \frac{1}{j1}} = \frac{1}{\sqrt{2}}\angle -45^\circ$$

$$i(t) = \cancel{0x} \quad 7.07 \sin(t - 45^\circ)$$

Q.

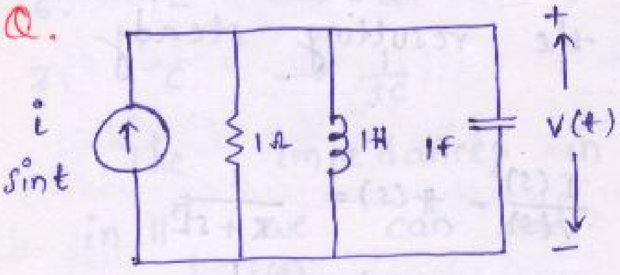


Determine the steady state voltage V_o

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{s}{s+1} = \frac{j\omega}{j\omega+1} = \frac{j1}{j+1}$$

$$\Rightarrow V_o = \frac{j1}{1+j} V_i$$

Q.



Determine the steady state voltage.

$$\frac{V(s)}{I(s)} = H(s) = Z(s) = \frac{1}{Y(s)} = \frac{1}{1/R + \frac{1}{sL} + sC}$$

$$\longrightarrow H(s) = \frac{1}{1 + \frac{1}{s} + s}$$

$$H(j\omega) = \frac{1}{1 + \frac{1}{j\omega} + j\omega}$$

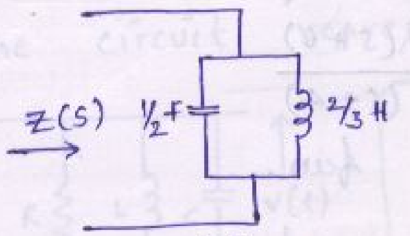
$$H(\omega)\Big|_{\omega=1} = 1$$

Network Synthesis :-

Q. The driving point impedance fun. of a 1-port n/w is $Z(s) = \frac{2s}{s^2+3}$. The realization is —?

$$Z(s) = \frac{2s}{s^2+3} = \frac{1}{\frac{s}{2} + \frac{3}{2s}} = \frac{1}{Y(s)}$$

$$Y(s) = s \cdot \tfrac{1}{2} + \tfrac{1}{s} \cdot \tfrac{2}{3} = sC + \frac{1}{sL}$$

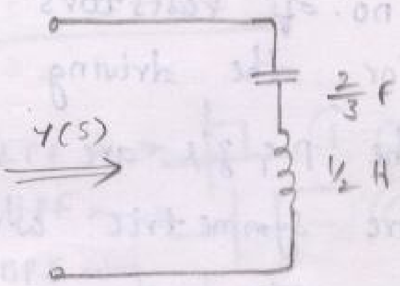


Q. $Y(s) = \frac{2s}{s^2+3}$

$$= \frac{1}{\frac{s}{2} + \frac{3}{2s}} = \frac{1}{Z_s}$$

$$Z(s) = s \cdot \frac{1}{2} + \frac{1}{s \cdot 2/3}$$

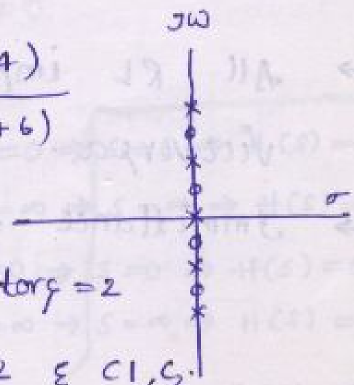
$$= sL + \frac{1}{sC}$$



NOTE:-

1. For the driving point LC impedance fun., the poles and zeros are alternate, lies only on the $j\omega$ axis and both the numerator and denominator polynomial decrease must be default by one.

Eg:- $Z(s) = \frac{(s^2+1)(s^2+4)}{s(s^2+2)(s^2+6)}$



No. of inductors = 2, no. of capacitors = 2

Realize the above fun. f1, f2 & C1, C2.

For the driving point RL fun. the P & Z are alternate and lies only on the -ve real axis and nearest to the origin is the zero.

[zero can be at the origin]

Eg:- Realize the $Z(S) = \frac{(S+1)(S+4)}{(S+2)(S+6)}$ by FF I, FF-II, C-I, C-II.

Q. For the driving point RC fun. the P & Z's are alternate, lies only on −ve real axis and nearest to the origin is the pole.

[pole can be at the origin].

eg:- $Z(S) = \frac{(S+2)(S+6)}{(S+1)(S+4)}$

Q. Realize the above fun.

no. of capacitors = 2

no. of resistors = 3.

For the driving point RLC impedance fun. the P & Z's are complex conjugate and they are symmetric w.r.t. the −ve real axis.

NOTE:- In the above cases instead of impedance fun.s, admittance fun. are given, then they are converted into impedance fun.s and above test can be performed.

→ All RL imp. fun = RC adm. fun. and viceversa.

→ Immittance = impedance (or) admittance

Filters:-

$Z_L = j\omega L\ \Omega$

$Z_C = \dfrac{1}{j\omega C}\ \Omega$

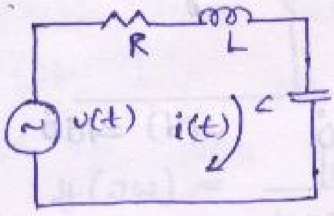
$\omega = 0, \Rightarrow Z_L = 0 \Rightarrow L \rightarrow SC$

$Z_C = \infty \Rightarrow C \rightarrow OC$

$\omega = \infty \Rightarrow Z_L = \infty \Rightarrow L \rightarrow \infty$

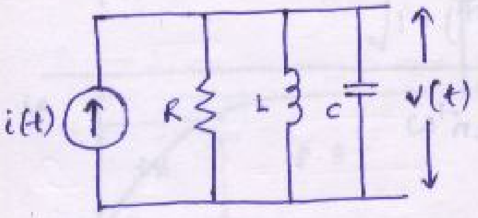
$Z_C = 0 \Rightarrow C \rightarrow 0$

Q.



$\omega = 0 \Rightarrow i(t) = 0$ & $\omega = \infty \Rightarrow i(t) = 0$
so the circuit represents a band pass filter.

Q. The circuit represent

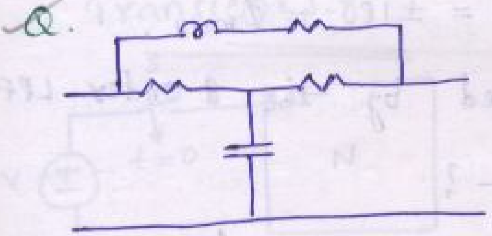


$\omega = 0 \Rightarrow L \rightarrow S.C. \Rightarrow v(t) = 0$
$(s \rightarrow \infty)$ $C \rightarrow O.C$ (BPF)

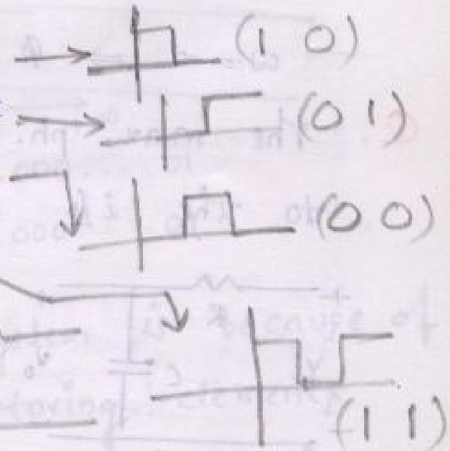
$\omega = \infty \Rightarrow L \rightarrow O.C$
$(s \rightarrow 0)$ $C \rightarrow S.C$ $v(t) = 0$

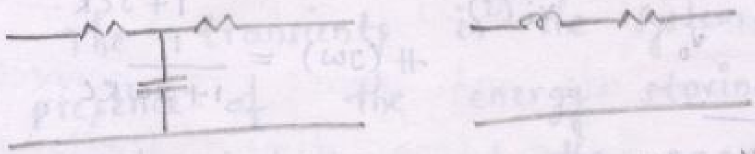
So the circuit is _

Q.



a). LPF (1 0)
b). HPF (0 1)
c). BPF (0 0)
d). BSF (1 1)





$\omega = 0 \Rightarrow V_0 = V_i$
$\omega = \infty \Rightarrow V_0 = 0$

$\omega = 0 \Rightarrow V_0 = V_i$
$\omega = \infty \Rightarrow V_0 = 0$

Second Order Filters :-

LPF $\Rightarrow H(S) = \dfrac{1}{S^2+S+1}$; $\omega = 0 \Rightarrow S = 0 \Rightarrow H(S) = 1$
$\omega = \infty \Rightarrow S = \infty \Rightarrow H(S) = 0$

HPF $\Rightarrow H(S) = \dfrac{S^2}{S^2+S+1}$; $\omega = 0 \Rightarrow S = 0 \Rightarrow H(S) = 0$
$\omega = \infty \Rightarrow S = \infty \Rightarrow H(S) = 1$

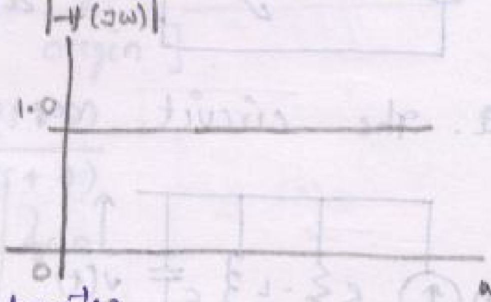
BPF $\Rightarrow H(S) = \dfrac{S}{S^2+S+1}$; $\omega = 0 \Rightarrow S = 0 \Rightarrow H(S) = 0$
$\omega = \infty \Rightarrow S = \infty \Rightarrow H(S) = 0$

BSF $\Rightarrow H(S) = \dfrac{S^2+1}{S^2+S+1}$; $\omega = 0 \Rightarrow S = 0 \Rightarrow H(S) = 1$
$\omega = \infty \Rightarrow S = \infty \Rightarrow H(S) = 1$

APF $\Rightarrow$

Q. The max. phase shift added by APF to the i/p signal is — ?

(a) 0° (b) 90° (c) -90° (d) $\pm 180^\circ$



$$H(s) = \frac{1-s}{1+s}$$

$$H(j\omega) = \frac{1-j\omega}{1+j\omega}$$

$$|H(j\omega)| = 1$$

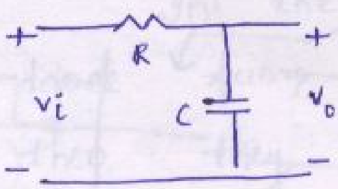
$$\angle H(j\omega) = \phi = -\tan^{-1}\omega - \tan^{-1}\omega$$

$$= -2\tan^{-1}\omega$$

$\omega = 0 \Rightarrow \phi = 0^\circ = \phi_{min}$

$\omega = \infty \Rightarrow \phi = -180^\circ = 180^\circ = \pm 180^\circ = \phi_{max}$.

Q. The max. ph. shift added by the I order LPF to the i/p signal is — ?



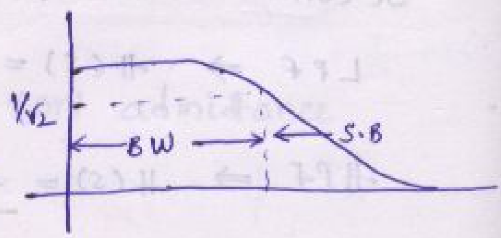
$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{1}{1+sCR}$$

$$H(j\omega) = \frac{1}{1+j\omega RC}$$

$$H(j\omega) = \frac{1}{1+j\,f/f_L} \quad \text{where } f_L = \frac{1}{2\pi RC}$$

$$|H(j\omega)| = \frac{1}{\sqrt{1+(f/f_L)^2}}$$

$$\angle H(j\omega) = \phi = -\tan^{-1} f/f_L$$



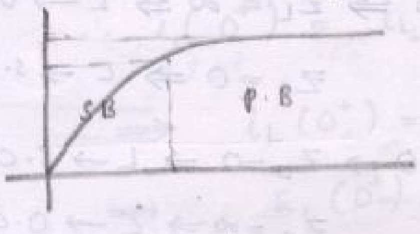
Q. The max. ph. shift added by I order HPF to the i/p signal is — ?

$$\frac{V_o(s)}{V_i(s)} = H(s) = \frac{sCR}{1+sCR}$$

$$H(j\omega) = \frac{1}{1+\frac{1}{j\omega CR}}$$

$$H(j\omega) = \frac{1}{1-j\,f_H/f} \quad \text{where } f_H = \frac{1}{2\pi RC}$$

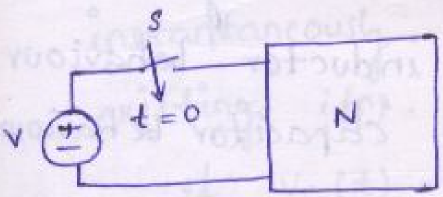
$$|H(j\omega)| = \frac{1}{\sqrt{1+(f_H/f)^2}} \; ; \; \angle H(j\omega) = \phi = +\tan^{-1}\frac{f_H}{f}$$



$f = f_H \Rightarrow \phi = 45^\circ = \phi_{max}$

$f =$

Transients:-



0^- 0^+

$-\infty$ 0 $+\infty$

$0^- = -0.000\ldots.01$

$0^+ = +0.000\ldots.01$

1. The transients in the system is because of presence of the energy storing elements called inductor and the capacitor. Since the energy stored in a memory element cannot change instantaneously [within zero time], the L, C elements will oppose the sudden changes in the system, which results the unstability in the n/w due to the oscillations.

If the next n/w consists of only resistors then no transients will results in the system at the time of switching, the resistor can accommodate any amount of V & I.

The transient effects are more severe in DC as compare with AC and the transient free time is possible only for AC.

1. The behaviour of L & C at $t = 0^+$ and as $t \to \infty$:-

$Z_L = sL\ \Omega$, $t = 0^+ \Rightarrow s = \infty \Rightarrow Z_L = \infty \Rightarrow L \to$ O.C.

$Z_C = 0 \Rightarrow C \to$ S.C

$t \to \infty \Rightarrow s \to 0 \Rightarrow Z_L = 0 \Rightarrow L \to$ S.C

$Z_C = \infty \Rightarrow C \to$ O.C.

→ A long time after the switching is nothing but the SS. In SS, the inductor behaviour is S.C. behaviour and the capacitor behaviour is O.C.

Steady state:-

whenever the ind. source is connected to the n/w for a long time ideally infinite amount of time, practically upto 5 time constants] then the n/w is said to be in the SS. In SS the energy stored in memory elements is max. and constant.

ie $\frac{1}{2} L i_L^2$ = max & const.

$\Rightarrow i_L$ = max & const.

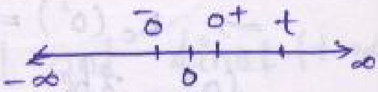
since $V_L = L \cdot \frac{di_L}{dt} \Rightarrow V_L = 0 \Rightarrow L \to$ S.C.

Similarly $\frac{1}{2} C V_C^2$ = max & const.

$\Rightarrow$ V_c = max & const.

since $i_c = C \cdot \frac{dV_c}{dt} \Rightarrow i_c = 0 \Rightarrow C \rightarrow$ O.C.

The inductor i and capacitor volt. at $t = 0^-$ and at $t = 0^+$ instants.



L:- $i_L(t) = \frac{1}{L}\int_{-\infty}^{t} V_L(t)\,dt$

$= \frac{1}{L}\int_{-\infty}^{0^-} V_L(t)\,dt + \int_{0^-}^{t} \frac{1}{L} V_L\,dt$

$= i_L(0^-) + \frac{1}{L}\int_{0^-}^{t} V_L(t)\,dt.$

$\therefore\ i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} \cancel{V_L(t)}\,dt \rightarrow 0$

$\Rightarrow\ i_L(0^+) = i_L(0^-)$

$\Rightarrow\ E_L(0^+) = E_L(0^-)$

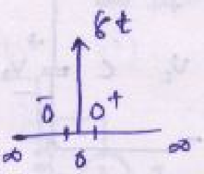
$\longrightarrow$ so, the inductor i cannot change instantaneously ie for all the practical existing i/ps. similarly the energy.

If $V_L(t) = \delta(t)$ then

$i_L(0^+) = i_L(0^-) + \frac{1}{L}\int_{0^-}^{0^+} \delta(t).\,dt \longrightarrow$ Area = 1.

$i_L(0^+) = i_L(0^-) + 1/L$

$\Rightarrow E_L(0^+) > E_L(0^-)$



$V_L(t) = \delta(t) = 0$ for $-\infty \leq t < 0^-$

so $i_L(0^-) = \frac{1}{L}\int_{-\infty}^{0^-} V_L(t).\,dt = 0\,A$

$\Rightarrow\ i_L(0^+) = 1/L$

so $E_L(0^-) = 0\,J$

$E_L(0^+) = 1/2\, L\,(1/L)^2 = \frac{1}{2L}\,(J)$

C:- $V_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\,dt$

$= V_c(0^-) + \frac{1}{C}\int_{0^-}^{t} i_c(t)\,dt$

At $t = 0^+$

$$V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} i_c(t)\,dt \rightarrow 0$$

$$\Rightarrow V_c(0^+) = V_c(0^-)$$

$$\Rightarrow E_c(0^+) = E_c(0^-)$$

So the capacitor volt. can't change instantaneously for all the practical i/p's. Similarly the energy.

If $i_c(t) = \delta(t)$, then $V_c(0^+) = V_c(0^-) + \frac{1}{C}\int_{0^-}^{0^+} \delta(t)\,dt$

$$\Rightarrow V_c(0^+) = V_c(0^-) + \frac{1}{C}$$

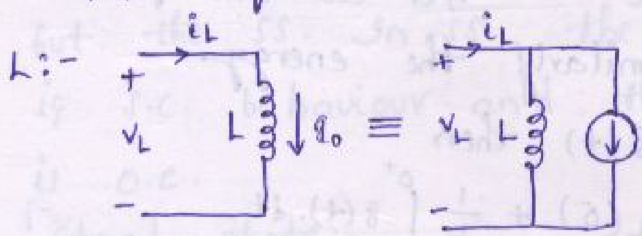
$$\Rightarrow E_c(0^+) > E_c(0^-)$$

$i_c(t) = \delta(t) = 0$ for $-\infty \le t < 0^-$

So $V_c(0^-) = \frac{1}{C}\int_{-\infty}^{0^-} i_c(t)\,dt = 0 \Rightarrow V_c(0^+) = 1/C$.

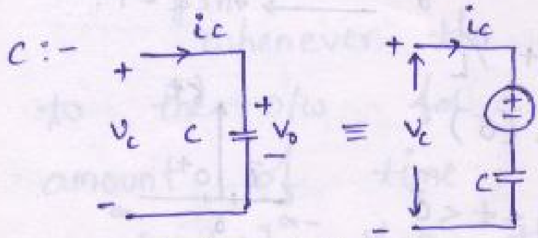
3. The equivalent circuits :-

L :-



$$i_L = \frac{1}{L}\int_{-\infty}^{t} V_L\,dt \equiv i_L = i_L(0) + \frac{1}{L}\int_0^t V_L\,dt \quad i_L(0) = I_0$$

C :-



$$V_c = \frac{1}{C}\int i_c(t)\,dt \equiv V_c = V_c(0) + \frac{1}{C}\int_0^t i_c\,dt \quad V_c(0) = V_0$$

Transients :-

- DC
- AC

DC transients :-

1. Source free circuits (without ind. sources)
   - RL
   - RC
   - RLC

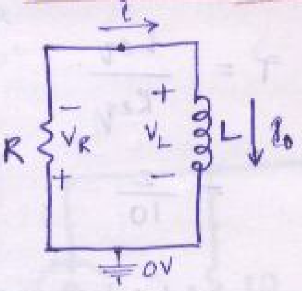
In all the source free ckts, the L and C will looses their energies to resistors as a fun. of time, hence the energy in the SS is always $= 0$.

2. With sources initial $[t = 0^+]$ and final $[t \to \infty]$ ~~instants~~ conditions :-
At these 2 instants, L and C elements will looses their significance and hence the nature of the ckt is resistive.

3. with sources the laplace transform approach of solving the transient problems for $t \geq 0$, $t \leq \infty$.

Source free circuits :-

1. Source free RL circuit :-



$$-V_R - V_L = 0$$

$$-iR - L.\frac{di}{dt} = 0$$

$$\Rightarrow \frac{di}{dt} + \frac{R}{L}.i = 0$$

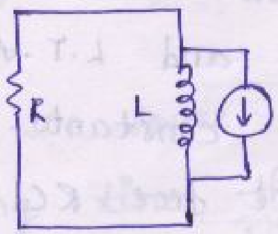
char. eq
$D + R/L = 0$

$$\Rightarrow Di + R/L\, i = 0 ,$$

$\Rightarrow D = -R/L$

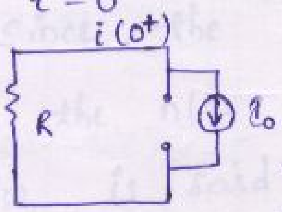
$$i(t) = k e^{\alpha t}, \; t \geq 0$$

$$= k . e^{-R/L\, t} , \; t \geq 0$$



for $t \geq 0$

At $t = 0^+$



$$-i(0^+) + I_0 = 0$$

$$\Rightarrow i(0^+) = I_0$$

$$i(0^+) = k e^{-0}$$

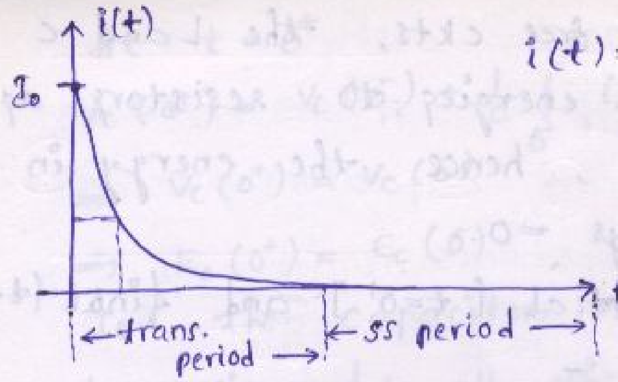
$$\Rightarrow k = i(0^+) = I_0$$

So $i(t) = I_0 \cdot e^{-R/L\, t}$, $0 \leq t \leq \infty$

$i(t) = I_0 e^{-t/\tau}$, $0 \leq t \leq \infty$

$\tau = L/R =$ sec $=$ time const. of the ckt.

$$V_L = L. \frac{di(t)}{dt}$$

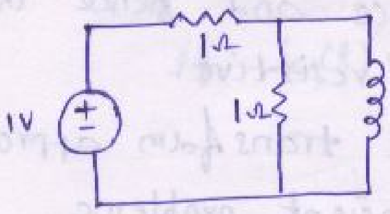


$$i(t) = I_0 e^{-5}$$

$$= \frac{I_0}{e^5} = 0.0067 I_0 < 1.1$$

$$\approxeq$$

Q. Determine the time constant of the circuit



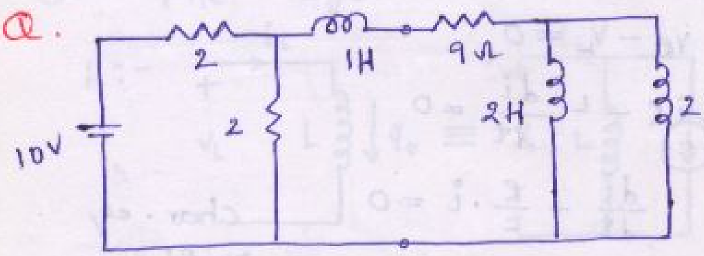
$$\tau = \frac{L}{R_{eq}}$$

$$= \frac{1}{1/2} = 2 \text{ sec}$$

Q. In the above problem, instead of vs if cs is present.

$$\tau = \frac{L}{R_{eq}} = 1/1 = 1 \text{ sec}$$

Q.

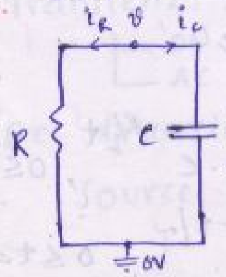


$$\tau = \frac{L_{eq}}{R_{eq}}$$

$$= \frac{2}{10}$$

NOTE: When the inductors and capacitors are not seperable then the circuit will have multiple τ's and L.T. Approach is used to evaluate these constants.

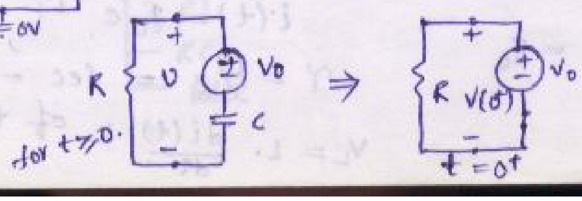
2. Source free RC :-



$$i_R + i_C = 0$$

$$\frac{v}{R} + C \cdot \frac{dv}{dt} = 0$$

$$\Rightarrow v(t) = K \cdot e^{-t/RC} \text{ for } 0 \le t \le \infty$$



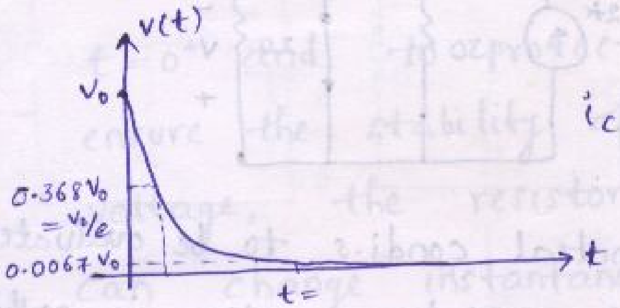
$$v(0^+) = V_0$$

$$v(0^+) = K e^{-0}$$

$$\Rightarrow K = v(0^+) = V_0$$

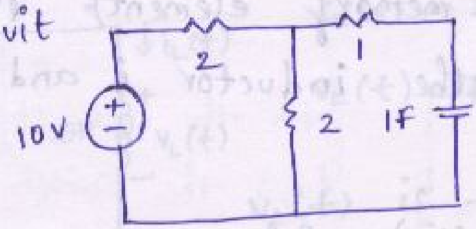
$v = v_0 e^{-t/RC}$, for $0 \le t \le \infty$

$v(t) = v_0 e^{-t/\tau}$, $\tau = RC$ = Time constant.



$i_c = C \cdot \frac{dv(t)}{dt}$

Q. Determine the time constant of the circuit



$\tau = R_{eq} \cdot C$

$= (1 + 2 \| 2) \cdot 1$

$= 2$ sec

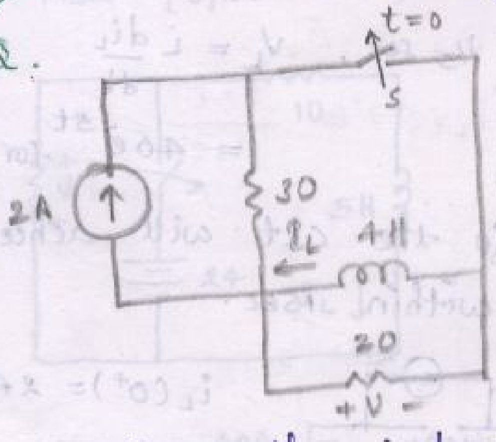
Q. when the vs is replaced by a cs of 10A then τ - ?

$\tau = R_{eq} \cdot C$

$= (1+2) \cdot 1 = 3$ sec.

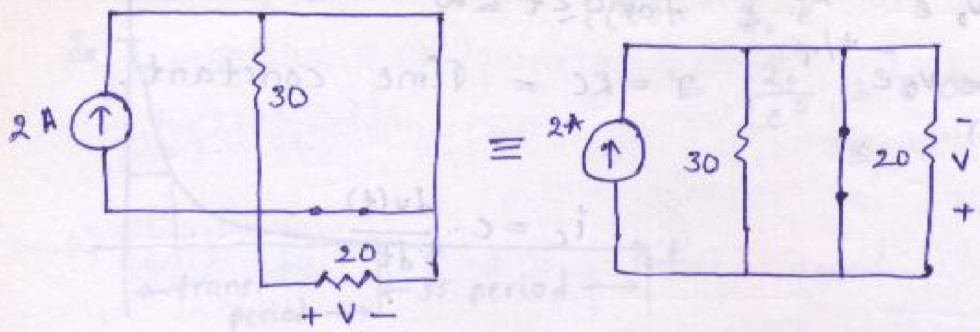
03-06-07

Q.



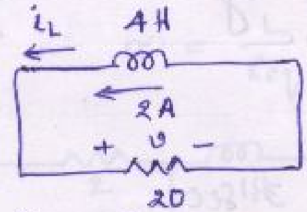
For fig. shown, the S is closed for a long time and it is opened t=0.

Since the independent DC source connected to the n/w for a long time then the n/w is said to be in SS. So the inductor acts as S/C. and the nature of the ckt is resistive.



The no. of initial condi.s to be evaluated at just before the switching action is nothing but the no. of memory elements present in the n/w ie the inductor i and capacitor voltage.

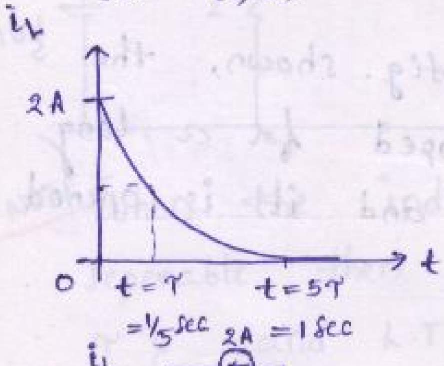
(i). $t = 0^-$, $i_L(0^-) = 2A$



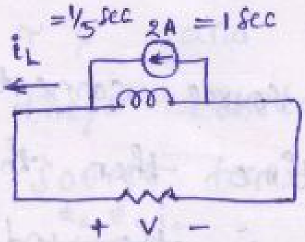
$$i_L(t) = 2 \cdot e^{-\frac{20}{4}t} \text{ for } t \geq 0$$
$$= 2 \cdot e^{-5t}$$

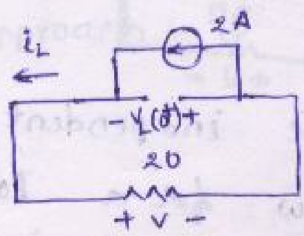
for $t \geq 0$,

$T = 1/5$ sec, $V_L = L\frac{di_L}{dt}$
$= 40e^{-5t}$ for $t \geq 0$



So the ckt will achieve ss within 1sec.





$i_L(0^+) = 2A$

$v(0^+) = 20 \times 2 = 40$ V

$t = 0^+$, $-V_L(0^+) - v(0^+) = 0$

$\Rightarrow V_L(0^+) = -v(0^+) = -40$ V

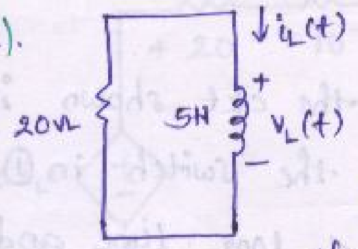
Obs:-

	$t = 0^-$	$t = 0^+$
1.	$i_L(0^-) = 2A$	$i_L(0^+) = 2A$
	$i_{20\Omega}(0^-) = 0A$	$i_{20\Omega}(0^+) = 2A$
	$V_{20\Omega}(0^-) = 0V$	$V_{20\Omega}(0^+) = 40V$

$V_L(0^-) = 0V \qquad V_L(0^+) = -40V$

Conslusion :- 1. To keep the energy const at $t=0^+$ and to protect KCL, KVL ie to ensure the stability of n/ω, the inductor voltage, the resistor i and its voltage can change instantaneously ie within 0 time at $t=0^+$.

(2).

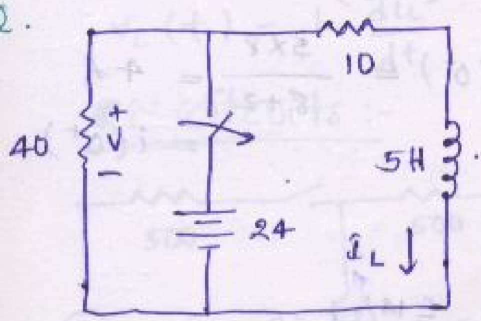


$i_L(t) = 2e^{-5t}$, for $t \geq 0$

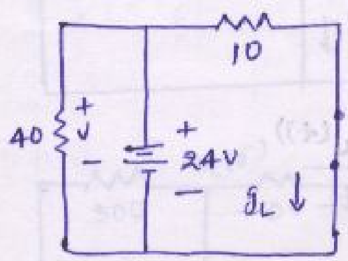
$V_L(t) = -40e^{-5t}$, for $t \geq 0$

$V_L(t)$ is −ve for all $t \geq 0$, because the inductor while acting as a temperary source, it discharges from the +ve terminal ie the i will enter at the −ve terminal. [upto 5τ]

Q.



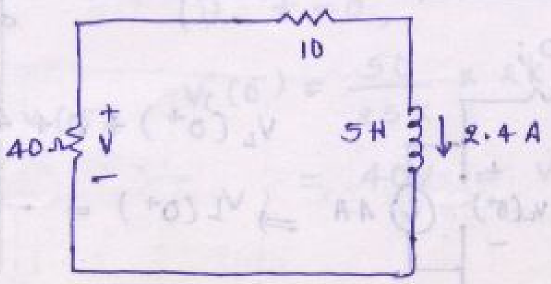
The s is closed for a long time and it is closed at $t=0$. Determine $i_L(0^-)$



$t=0^-$, SS.

(i). $t=0^-$,

$$i_L(0^-) = \frac{24}{10} = 2.4\ A = i_L(0^+)$$



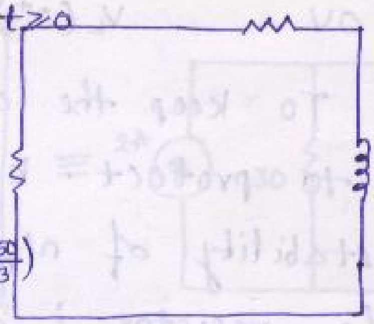
for $t \geq 0$

$i_L(t) = 2.4e^{-\frac{50}{5}t}, \; t \geq 0$

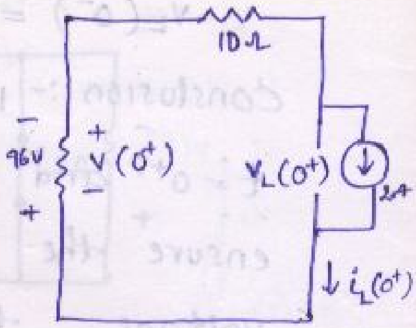
$\tau = \frac{1}{10}$ sec

$v_L(t) = L \cdot \frac{di_L(t)}{dt}$

$= 5(2.4)e^{-\frac{50}{5}t} \cdot \left(-\frac{50}{3}\right)$



$t \geq 0$



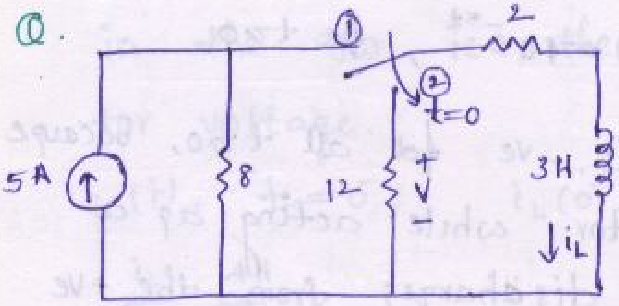
$t = 0^+$

$v(0^+) = -96$ V

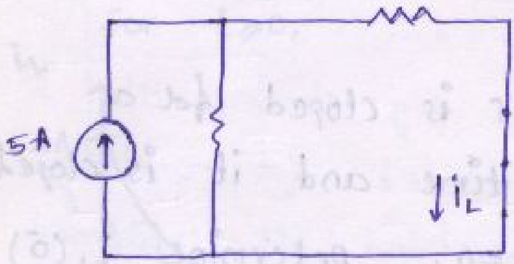
$v_L(0^+) = -96 - 24$

$= -120$ V

Q. For the ckt shown in fig. the switch in ① for a long time and it is moved ② at t=0.



Determine $i_L(0^+)$, $v_L(0^+)$ & $i_L(t)$ for $t > 0$.

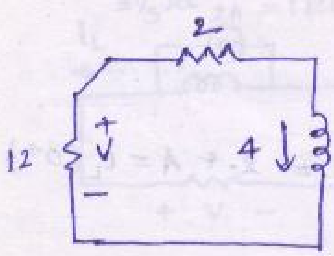


$t = 0^-$; SS.

(i). $t = 0^-$,

$i_L(0^-) = \frac{5 \times 8}{8+2} = 4A$

$= i(0^+)$

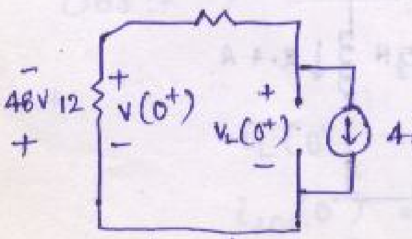


$t \geq 0$;

$i_L(t) = 4e^{-14/3\, t}, \; t \geq 0$

$\tau = 3/14$ sec

$v_L(t) = L \cdot \frac{d_L(i_L(t))}{dt}$

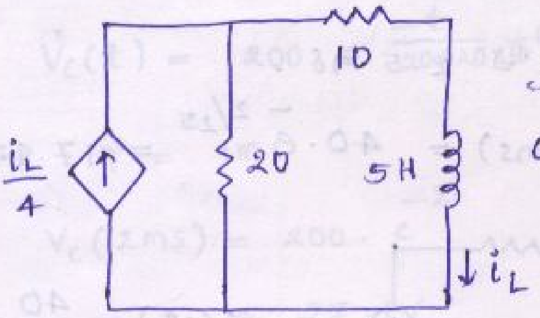


$t = 0^+$

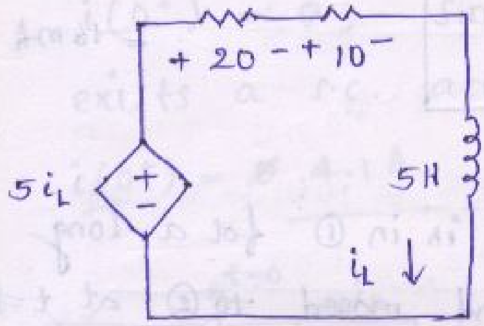
$v_L(0^+) + 8 + 48 = 0$

$\Rightarrow v_L(0^+) = -56$ V

Q. Determine the time constant of the circuit. Assume $i_L(0) = 10\,A$



It is a source free with RL.



$$5i_L - 20i_L - 10i_L - 5\cdot\frac{di_L}{dt} = 0$$

$$\frac{di_L}{dt} = -5i_L$$

$$\therefore i_L(t) = k\cdot e^{-5t}, \quad t \geq 0$$

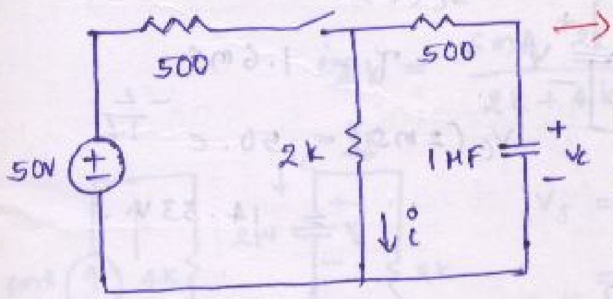
$$\tau = 1/5 \text{ sec}$$

$$i_L(0) = k e^{-0} \Rightarrow k = 10$$

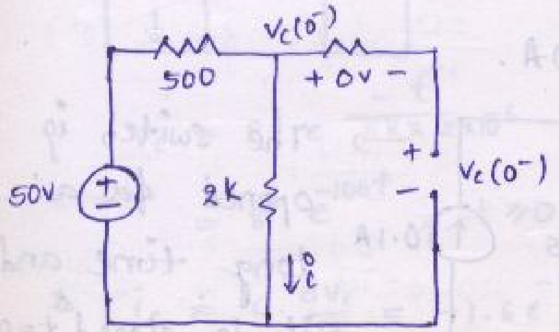
$$i_L(t) = 10\cdot e^{-5t}, \quad t \geq 0$$

$$v_L(t) = L\cdot\frac{di_L}{dt}$$

RC circuits :-



→ Switch is closed for a long time and it is opened at $t=0$. Determine $v_c(0^+)$, $i_L(0^+)$, $v_c(2\,msec)$

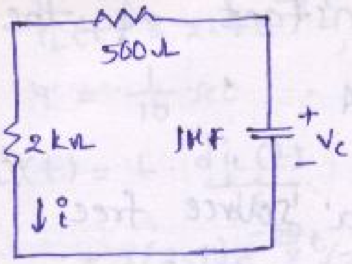


$t = 0^-$; SS

(i). $t = 0^-$,

$$v_c(0^-) = \frac{50}{2.5k} \times 2k$$

$$= 40V = v_c(0^+)$$



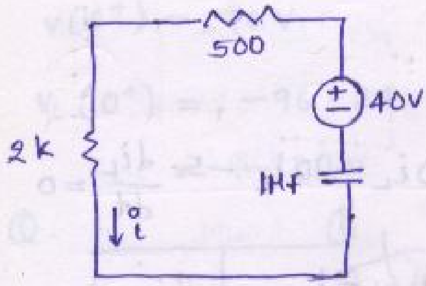
$t \geq 0.$

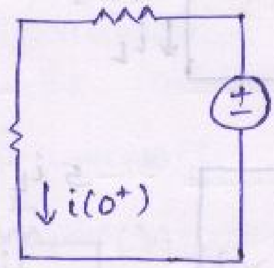
$v_c(t) = v_0 e^{-t/RC}$

$= 40 \cdot e^{-t/2.5\times10^{-3}}$

$\tau = 2.5\,ms.$

$v_c(2ms) = 40 \cdot e^{-2/2.5} = 17.97\,V$



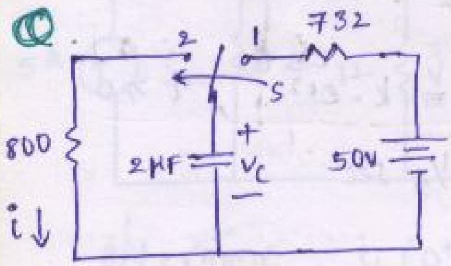


$t = 0^+$

$i(0^+) = \frac{40}{2.5k}$

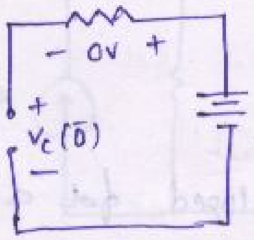
$= 16\,mA$

Q.



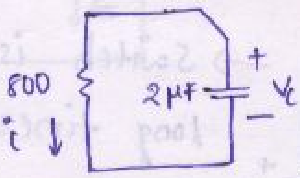
The s is in ① for a long time and moved to ② at t=0. Determine $v_c(0^+)$, $i(0^+)$ and $v_c(2ms)$.

(i). $t = 0^-$



$t = 0^-$, SS

$v_c(0^-) = 50V = v_c(0^+).$



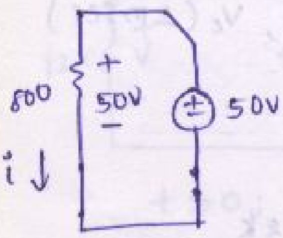
$t \geq 0$

$v_c(t) = 50 \cdot e^{\frac{-t}{1.6\times10^{-3}}}$

$\tau = 1.6\,ms$

$v_c(2ms) = 50 \cdot e^{-\frac{2}{1.6}}$

$= 14.33\,V.$

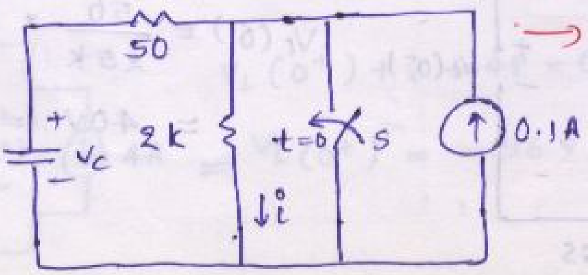


$t = 0^+$

$i(0^+) = \frac{50}{800}$

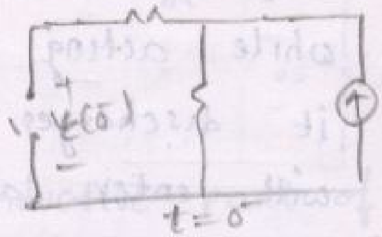
$= 62.5\,mA.$

Q.



→ The switch is opened for a long time and it is closed at t=0.

Determine $v_c(0^+)$, $i_c(0^+)$ and $v_c(2ms)$.



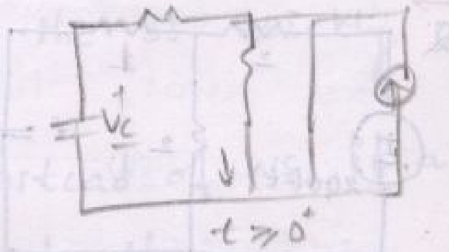
$v_c(0^-) = 200V = v_c(0^+)$

$v_c(t) = 200 \cdot e^{\frac{-t}{50\times20\times10^{-6}}}$

$\tau = 1ms$

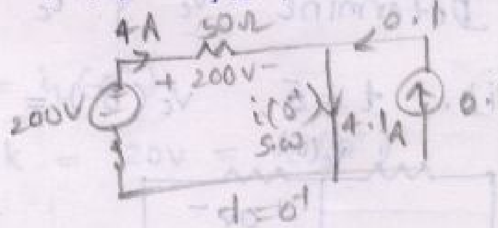
$v_c(2ms) = 200 \cdot e^{-2}$

$= 27.1V$

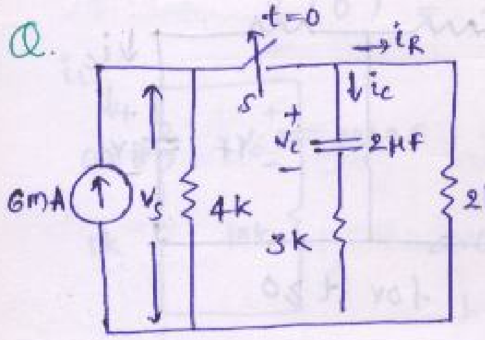


$i(0^+) = 0$, since there exists a s.c. across $2k\Omega$. from $t=0$ onwards

$i(0^+) = \cancel{0}\ 4.1A$



Q.

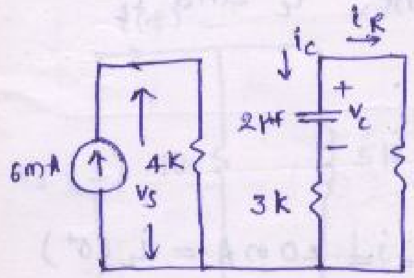


→ Determine $v_c(t)$, $v_s(t)$, $i_c(t)$, $i_R(t)$.

(i). $t=0^-$, $v_c(0^-) = V_{4k\Omega} = V_{2k\Omega}$

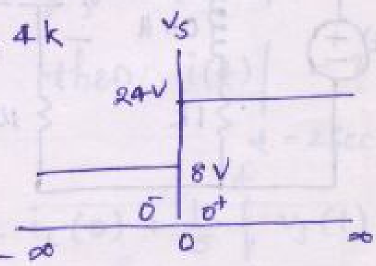
$= V_{4k\Omega} \Rightarrow i_{4k} \times 4k$

$i_{4k} = \frac{6mA \cdot 2k}{2k+4k}$



$V_S = 6mA \times 4k$

$= 24\ V$



$V_c = 8 \cdot e^{\frac{-t}{5k \times 2\times10^{-6}}}$

$= 8 \cdot e^{-100t}$, $t \geq 0$

$i_c + i_R = 0$

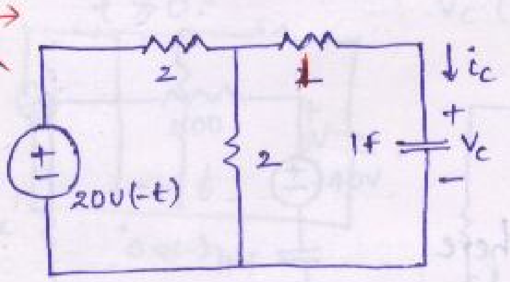
$i_R = -i_c$

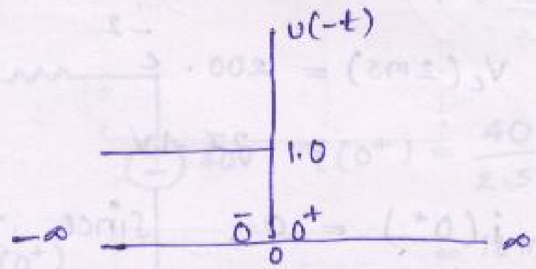
$i_c = c \cdot \frac{dv_c}{dt} = -1.6e^{-100t}$, $t \geq 0$ $\qquad = 1.6e^{-100t}$, $t \geq 0$.

Obs:-

i_c is $-ve$ $\forall$ $t \geq 0$. because the capacitor while acting as a temperary source upto 5τ it discharges from the $+ve$ terminal ie the i will enter at the $-ve$ terminal.

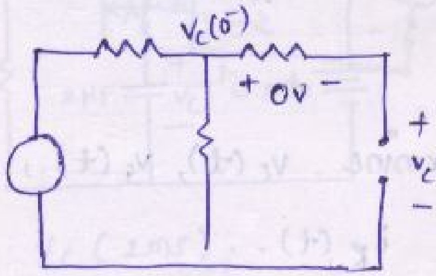
Q

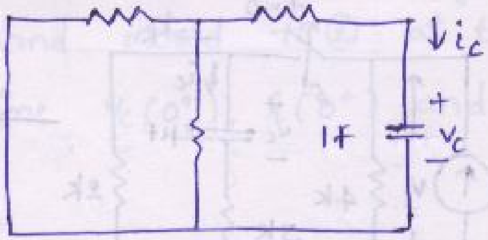




Determine V_c & i_c for $t > 0$.

(i). $t = \bar{0}$, $V_c(\bar{0}) = \frac{20}{4} \times 2 = 10V = V_c(0^+)$.





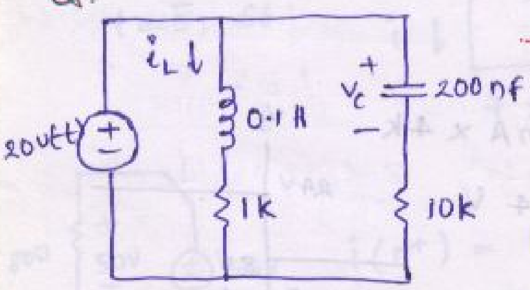
$t = \bar{0}$; SS

for $t \geq 0$

$V_c = 10\, e^{-t/2}, \; t \geq 0$

$i_c = c\frac{dV_c}{dt} = -5 \cdot e^{-t/2}, \; t \geq 0$.

Q.

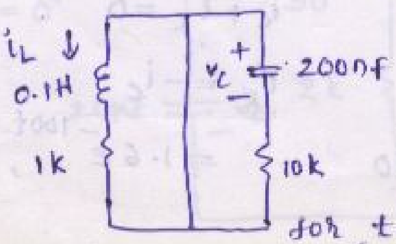


Determine i_c and i_L for $t > 0$.

(i). $t = \bar{0}$,

$i_L(\bar{0}) = \frac{20}{1K} = 20\,mA = i_L(0^+)$

$V_c(\bar{0}) = 20V = V_c(0^+)$.



for $t > 0$;

$i_L = 20 \cdot e^{-\frac{1 \times 10^3}{0.1}t}\; mA, \; t \geq 0$

$V_c = 20 \cdot e^{\frac{-t}{10k \times 200n}}\; V, \; t \geq 0$

$\tau_L = 100\,\mu s, \quad \tau_c = 2\,ms$

$V_L = L \cdot \frac{di_L}{dt}$ $\quad i_c = C \cdot \frac{dv_c}{dt}$

$\frac{\tau_C}{\tau_L} = 20 \Rightarrow \tau_C = 20\,\tau_L$.

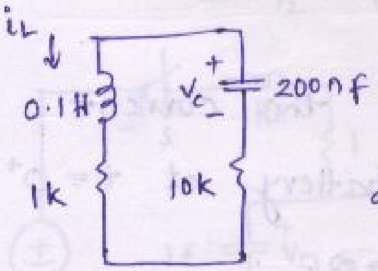
Obs:- Since $\tau_L < \tau_C$, the inductive part of the circuit will achieve the SS quickly ie 20 times faster.

In the above problem instead of VS, if a CS of $20\,u(-t)$ (↑) is used then

(i). $t = 0^-$

$$i_L(0^-) = 20\text{mA} = i_L(0^+)$$

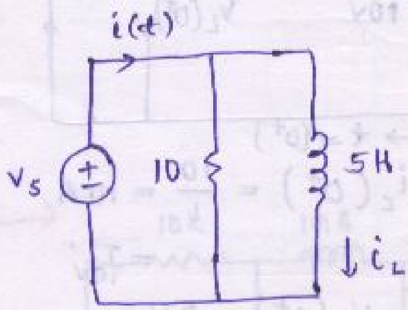
$$v_c(0^-) = 20\text{m} \times 1\text{k} = 20\text{V} = V_c(0^+)$$



$t > 0$.

NOTE:- The responses in the RLC, are generally evaluated by using L.T. Approach., since the II order differential eq. in the time domain ie the complex poles.

$$S_1, S_2 = \alpha \pm i\beta$$



If $V_s = 40t$ for $t \geq 0$ & $i_L(0) = 5A$ then $i(t)\big|_{t = 2sec} = ?$

$$i(t) = \frac{V_s(t)}{10} + i_L(0) + \frac{1}{5}\int_0^t V_s(t)\,dt$$

$$= 29\ A.$$

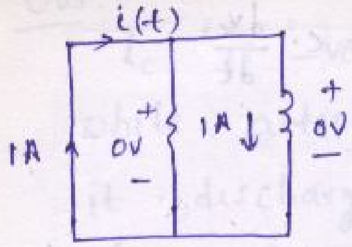
$$i(t) = \frac{\delta(t)}{1} + \frac{1}{1}\int_{-\infty}^{t} \delta(t)\,dt$$

$$= \delta(t) + u(t)$$

for impulse,

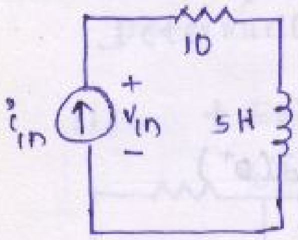
$i_L(0^-) = 0A$

$i_L(0^+) = \frac{1}{L} = 1A$



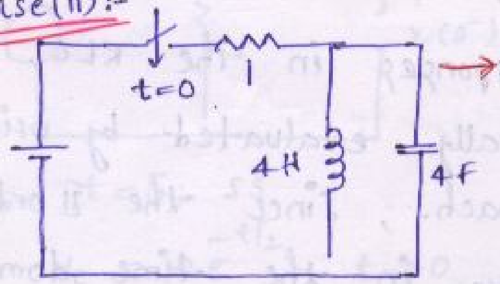
$$i(\alpha) = \delta(\infty) + u(\infty)$$
$$= 0 + 1 = 1A.$$
$$i(\infty) = i_L(\infty) = 1A = i(0^+) = i_L(0^+).$$

Obs:- Eventhough the circuit is a source free circuit, the energy in SS is non zero, because the inductor i is experiencing zero resistance for all $t > 0$.



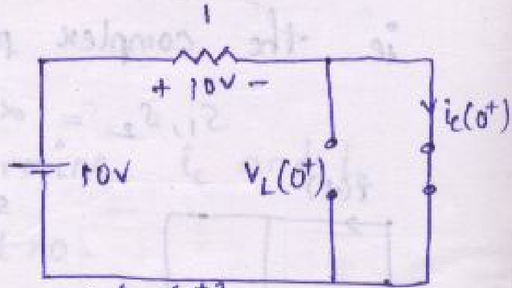
If $i_{in} = 0.4t^2$ for $t \geq 0$, then $v_{in}(t)\Big|_{t=1\,sec} = ?$ Ans: 8V

case(ii):-



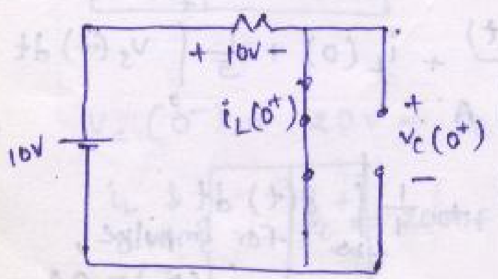
→ Determine the current through battery at $t = 0^+$ and $t \to \infty$.

$$i_L(0^-) = 0 = i_L(0^+).$$
$$V_c(0^-) = 0 = V_c(0^+)$$
$$E_L(0^-) = 0J = E_L(0^+)$$
$$E_c(0^-) = 0J = E_c(0^+)$$



→ $t = [0^+]$

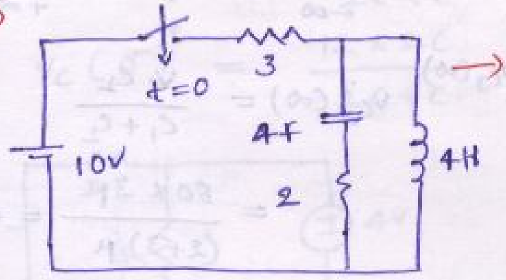
$$i_c(0^+) = \frac{10}{1} = 10A$$
$$= I_{10V}$$
$$V_L(0^+) = 0V$$



$t \to \infty$, SS.

$$V_L(\infty) = 0V$$
$$i_L(\infty) = 10/1 = 10A = I_{10V}$$
$$E_T(\infty) = E_L(\infty) + E_C(\infty)$$
$$= \frac{1}{2}L\,i_L(\infty)^2 + \frac{1}{2}C\,V_L^2(\infty)$$

$$= \tfrac{1}{2} \times 4 \times 10^2 + \tfrac{1}{2} \times 4 \times 10^2 = 200\,J.$$



→ The voltage across the inductor at $t = 0^+$.

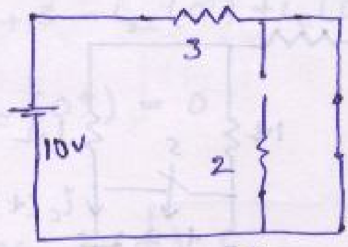
$$V_L(0^+) = 2 \times 2 = 4V.$$

$$i_L(0^-) = 0A = i_L(0^+)$$

$$V_C(0^-) = 0V = V(0^+)$$

$$E_L(0^-) = 0J = E_L(0^+)$$

$$E_C(0^-) = 0J = E_C(0^+)$$

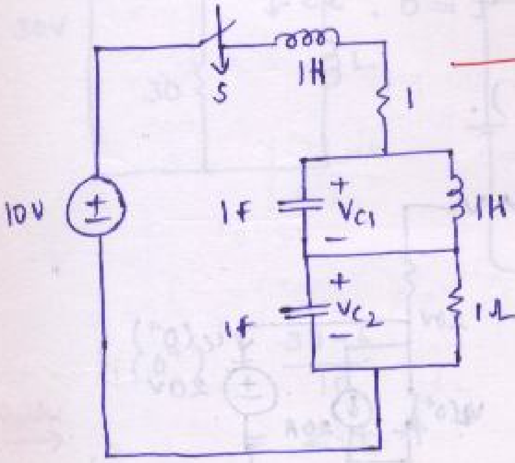


$t = \infty$, SS.

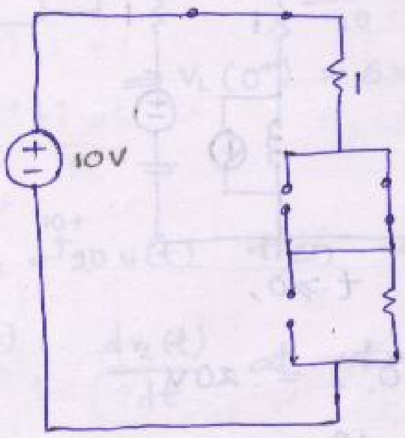
$\cancel{t=\infty}$

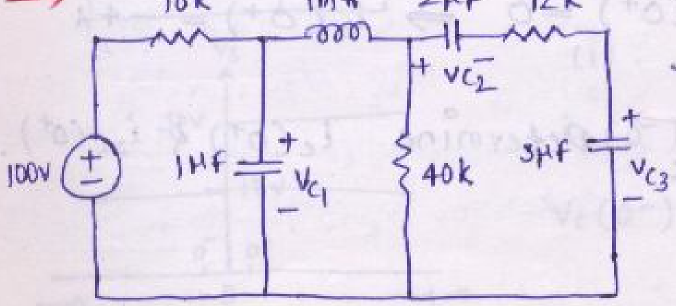
$$E_T(\infty) = E_L(\infty) + E_C(\infty)$$

$$= \tfrac{1}{2} \times 4 \times (10/3)^2 + \tfrac{1}{2} \times 4 \times 0^2 = \frac{200}{9}\,J.$$

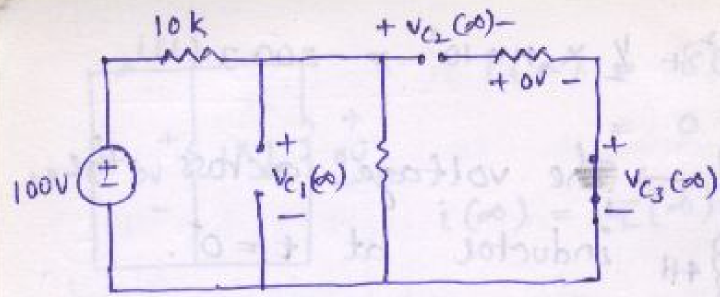


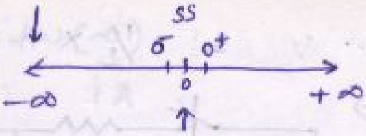
→ The SS. voltages across the capacitors are — ?





Determine the SS. voltages across the capacitors — ?

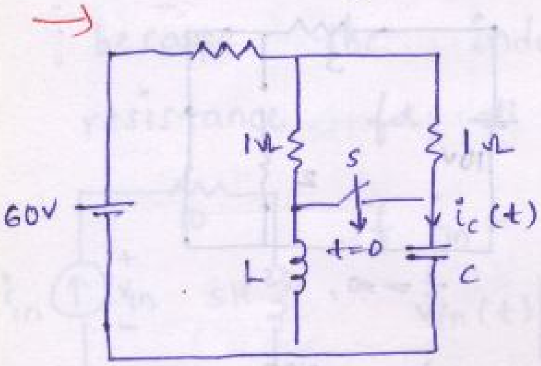


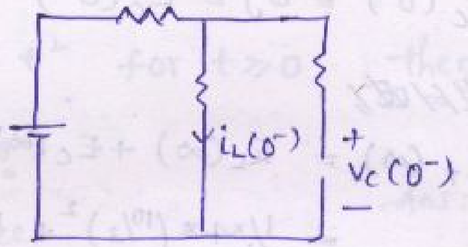


$$V_{C_2}(\infty) = \frac{V \cdot C_2}{C_1 + C_2} = \frac{80 \times 3\mu}{(2+3)\mu} = 48\,V$$

$t = \infty$; SS.

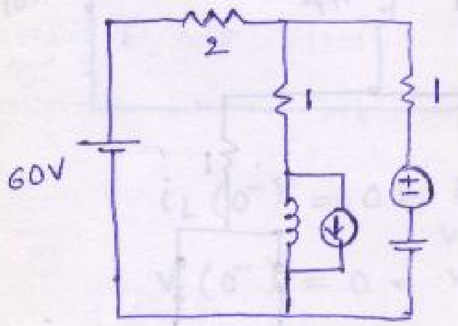
$$V_{C_1}(\infty) = \frac{100}{50} \times 40 = 80\,V \qquad V_{C_3}(\infty) = \frac{V \cdot C_1}{C_1 + C_2} = 32\,V$$



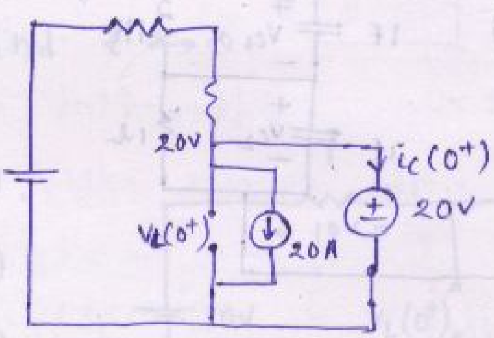


$$i_L(0^-) = \frac{60}{3} = 20A = i_L(0^+) \qquad t = 0^-,\ SS.$$

$$V_c(0^-) = 1 \times 20 = 20\,V = V_c(0^+).$$



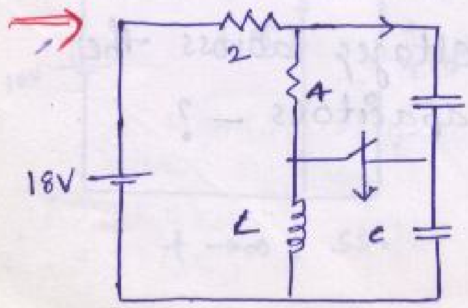
$t \geq 0$,



$t = 0^+$

$$V_L(0^+) = 20V$$

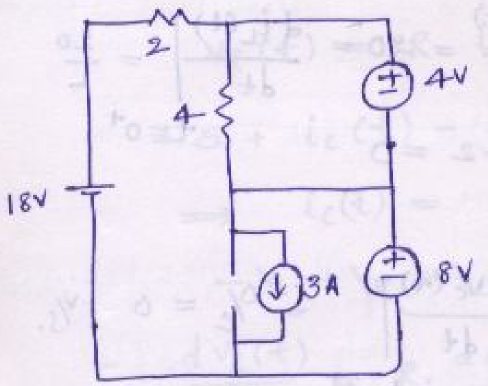
$$\frac{20 - 60}{2.5} + 20 + i_c(0^+) = 0 \Rightarrow i_c(0^+) = -4A$$



→ Determine $i_c(0^+)$ & $i_{2c}(0^+)$.

$$V_{2C}(0^-) = \frac{12 \times C}{2C + C} = 4V = V_{2C}(0^+)$$

$$V_C(0^-) = \frac{12 \times 2C}{2C + C} = 8V = V_C(0^+)$$



$$\frac{12-18}{2} + \frac{12-8}{4} + i_{2C}(0^+) = 0$$

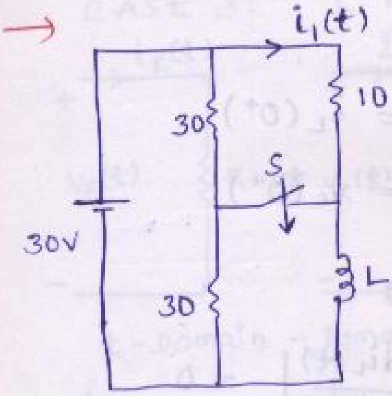
$$\Rightarrow i_{2C}(0^+) = 2$$

$$\frac{8-12}{4} + 3 - i_{2C}(0^+) + i_C(0^+) = 0$$

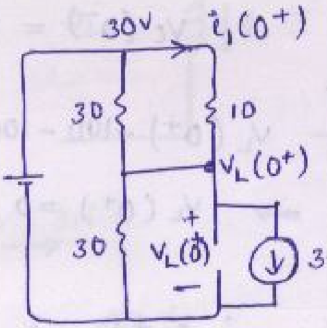
$$\Rightarrow i_C(0^+) = 0$$

$t = 0^+$;

→ Determine $i_1(0^+)$.



$i_1(0^-) =$



$t = 0^+$,

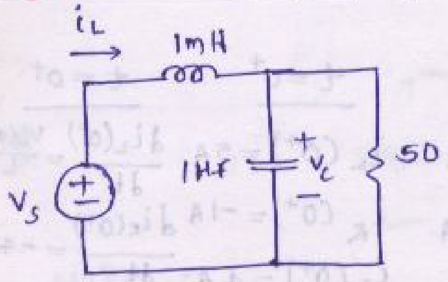
$$i_1(0^+) = \frac{30 - V_L(0^+)}{10}$$

$$\frac{V_L(0^+) - 30}{30} + \frac{V_L(0^+)}{30} + \frac{V_L(0^+) - 30}{10} + 3 = 0$$

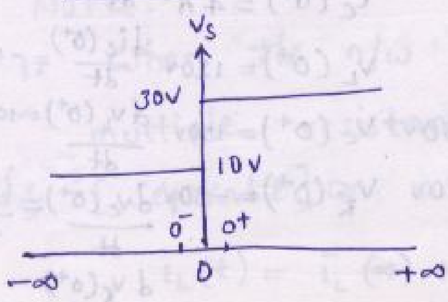
$$\Rightarrow V_L(0^+) = 6V$$

$$i_1(0^+) = \frac{30-6}{10} = 2.4\,A.$$

Problem →



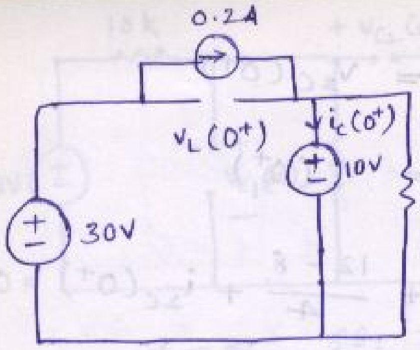
If $V_S = 10 + 20\,u(t)$ then find $\frac{di_L(t)}{dt}$, $\frac{dv_C(t)}{dt}$ at $t = 0^+$.



(i). $t = 0^-$,

$$i_L(0^-) = \frac{10}{50} = 0.2 = i_L(0^+)$$

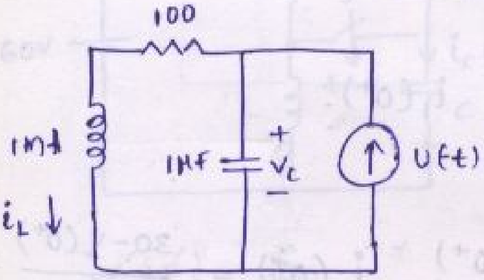
$$V_C(0^-) = 10V = V_C(0^+)$$



$t = 0^+$

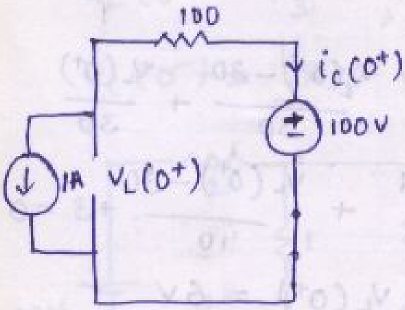
$$30 - v_L(0^+) - 10 = 0$$
$$\Rightarrow v_L(0^+) = 20$$
$$\Rightarrow v_L(t)\Big|_{t=0^+} = 20$$
$$L\,\frac{di(t)}{dt} = 20 = \frac{di_L(t)}{dt}\Big|_{t=0^+} = \frac{20}{L}$$

$$-0.2 + i_c(0^+) + 0.2 = 0$$
$$\Rightarrow i_c(0^+) = 0$$
$$i_c(t)\Big|_{t=0^+} = 0 \Rightarrow \frac{d\,v_c(t)}{dt}\Big|_{t=0^+} = 0/c = 0 \ \ V/s.$$



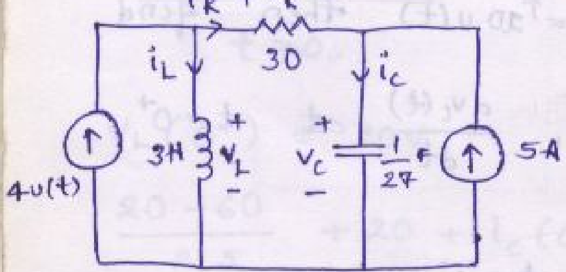
(i). $t = 0^-$

$$i_L(0^-) = 1\,A = i_L(0^+)$$
$$v_c(0^-) = 100 = v_c(0^+)$$



$t = 0^+$

$$v_L(0^+) + 100 - 100 = 0$$
$$\Rightarrow v_L(0^+) = 0, \quad \frac{di_L(t)}{dt}\Big|_{t=0^+} = 0$$
$$1 + i_c(0^+) = 0 \Rightarrow i_c(0^+) = -1$$
$$\frac{dv_c(t)}{dt}\Big|_{t=0^+} = -1/c$$



$t=0^-$	$t=0^+$	$t=0^+$
$i_L(0^-) = 5A$	$i_L(0^+) = 5A$	$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} =$
$i_R(0^-) = -5A$	$i_R(0^+) = -1A$	$\frac{di_R(0^+)}{dt} = -40\frac{A}{s}$
$i_c(0^-) = 0$	$i_c(0^+) = 4A$	$\frac{di_c(0^+)}{dt} = -40\frac{A}{s}$
$v_L(0^-) = 0$	$v_L(0^+) = 120V$	$\frac{dv_L(0^+)}{dt} = -1092\frac{V}{s}$
$v_R(0^-) = -150V$	$v_c(0^+) = 150V$	$\frac{dv_c(0^+)}{dt} = \frac{i(0^+)}{c}$
$v_c(0^-) = 150V$	$v_R(0^+) = -30V$	$\frac{dv_R(0^+)}{dt} = -1200\frac{V}{s}$

$t = 0^-$,

$$i_L(t) + i_R(t) = 0$$
$$\Rightarrow i_R(t) = -i_L(t),$$
$$\Rightarrow i_R(0^-) = -i_L(0^-)$$

$$v_R(t) = R \cdot i_R(t)$$

$$V_c(t) + V_R(t) - V_L(t) = 0$$

$\underline{t = 0^+}$, $-4 + i_L(t) + i_R(t) = 0$

$$\Rightarrow \quad i_R(0^+) = 4 - 5 = -1\,A$$

$$V_R(t) = R \cdot i_R(t)$$

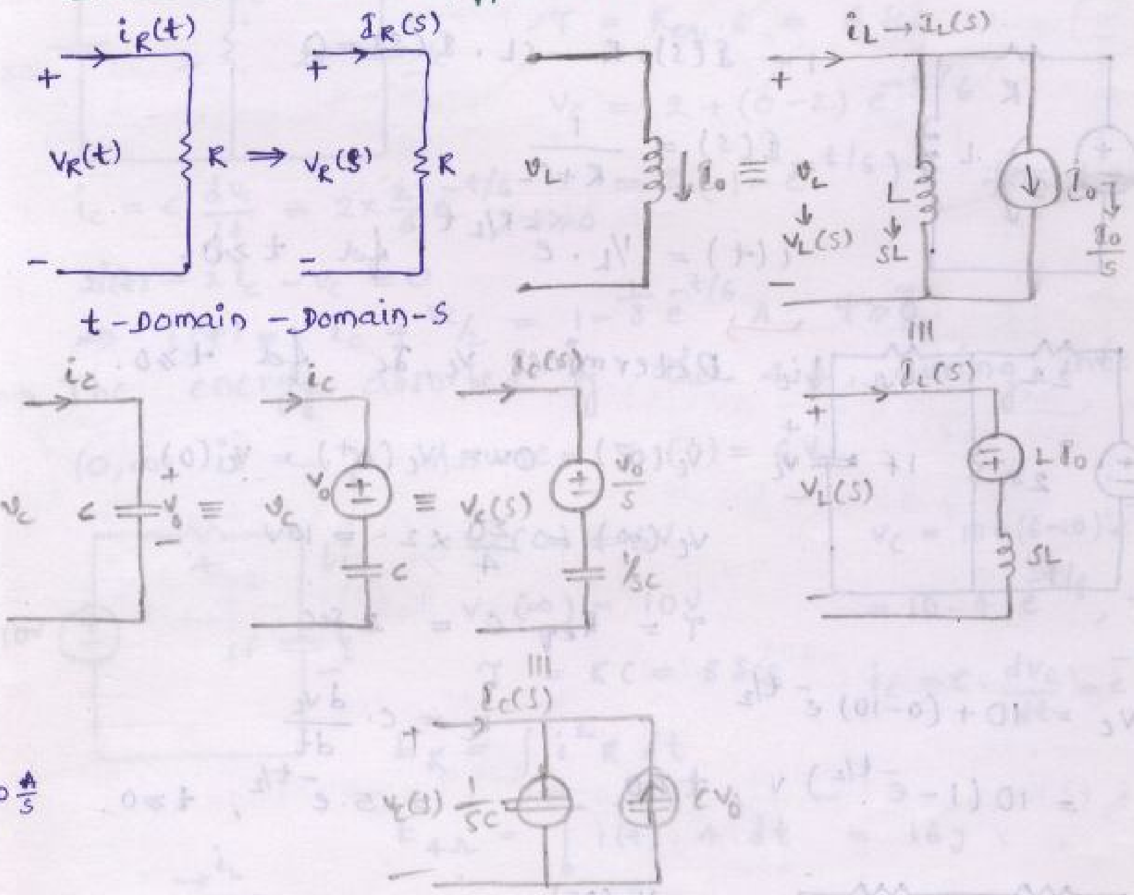
$$-5 + i_C(t) - i_R(t) = 0$$

$$\Rightarrow \quad i_C(t) = 5 + i_R(t)$$

$$-V_L + V_R + V_C = 0 \Rightarrow V_L = V_R + V_C$$

$$\frac{d V_R(t)}{dt} = R \cdot \frac{d i_R(t)}{dt}$$

CASE 3: L.T. Approach:-



NOTE :-

1. when the n/w consists of several ind. sources, multiple resistances and several inductances [seperable] or a single inductor then

$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]\, e^{-t/\tau}, \text{ for } 0 \le t \le \infty$$

$\tau = \frac{L}{R}$
$= \frac{L_{eq}}{R_{eq}}$ | $v_L(t) = L \cdot \frac{di_L(t)}{dt}$ | Obs: for source free circuity, $i_L(\infty) = 0$

$i_L(t) = i_L(0) \cdot e^{-t/\tau}$ for $0 \le t \le \infty$

2. In the above case instead of inductors, capacitors are present.

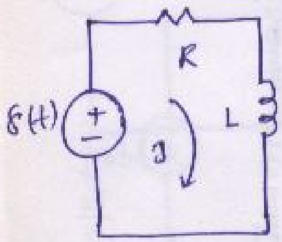
$v_c(t) = v_c(\infty) + (v_0(0) - v_c(\infty)) e^{-t/\tau}$ for $0 \le t \le \infty$

$\tau = RC$
$= R_{eq} C_{eq}$ | $i_c(t) = C \cdot \frac{dv_c(t)}{dt}$

Obs: for source free circuity $v_c(\infty) = 0$

$\Rightarrow v_c(t) = v_c(0) \cdot e^{-t/\tau}$ for $0 \le t \le \infty$

→ Determine $i(t)$, $t \ge 0$.

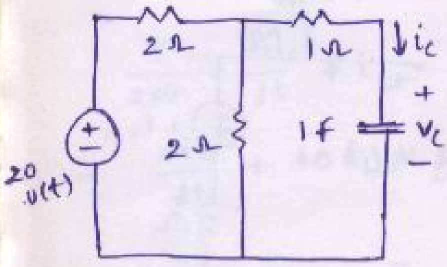


$1 - I(s) \cdot R - sL \cdot I(s) = 0$

$I(s) = \frac{1}{R + sL}$

$i(t) = \frac{1}{L} \cdot e^{-R/L\, t}$ for $t \ge 0$.

→ Determine v_c, i_c for $t \ge 0$.



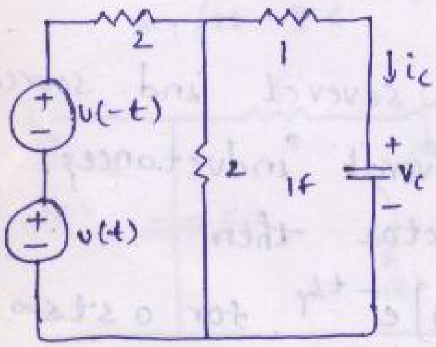
$v_c(0^-) = 0V = v_c(0^+) = v_c(0)$

$v_c(\infty) = \frac{20}{4} \times 2 = 10V$

$\tau = R_{eq} \cdot C = 2\ sec$

$v_c = 10 + (0 - 10) e^{-t/2}$
$= 10(1 - e^{-t/2})\ V$, $t \ge 0$

$i_c = C \cdot \frac{dv_c}{dt}$
$= 5 \cdot e^{-t/2}$, $t \ge 0$.

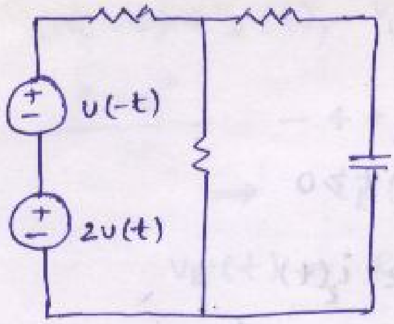


$v_c(0^-) =$

$v(\infty) = \frac{1}{4} \times 2 = \frac{1}{2} V$

$\tau = 2\ sec$

$v_c = \frac{1}{2} + (\frac{1}{2} - \frac{1}{2}) e^{-t/2}$



$v_c(0^-) = \frac{1}{4} \times 2 = \frac{1}{2} V = v_c(0^+)$

$v_c(\infty) = \frac{2}{4} \times 2 = 1V$

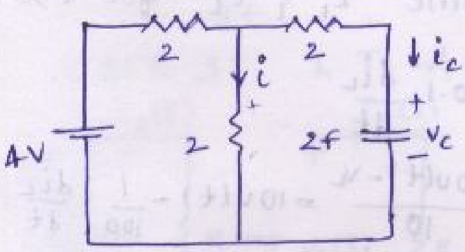
$\tau = 2\,m$

$v_c = 1 + (\frac{1}{2} - 1)\, e^{-t/2}$.

$= 1 - \frac{1}{2} e^{-t/2}, \quad t \geq 0$

$i_c = C \cdot \frac{dv_c}{dt} = \frac{1}{4} e^{-t/2}, \quad t \geq 0$

→ Determine v_c, i_c and $i(t)$ for $t > 0$. Assume $v_c(0) = 0V$



$v_c(0) = 0$

$v_c(\infty) = \frac{4}{4} \times 2 = 2V$

$\tau = R_{eq} \cdot C = 6\ sec$

$v_c = 2 + (0 - 2)\, e^{-t/6}$

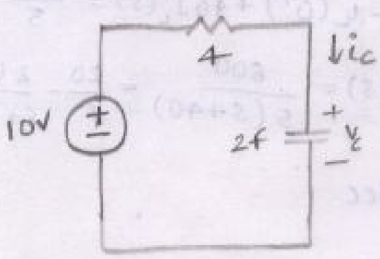
$= 2(1 - e^{-t/6})\ V$ for $t \geq 0$

$i_c = C\frac{dv_c}{dt} = 2 \times \frac{2}{6} e^{-t/6}, \quad t \geq 0$

$2i(t) - 2i_c - v_c = 0$

$\Rightarrow i(t) = i_c + v_c/2 = 1 - \frac{1}{3} e^{-t/6}\ A, \quad t \geq 0.$

→ The energy absorbed by the 4Ω, during interval $(0,\infty)$ is —? Assume $v_c(0) = 6V$.



$v_c(0) = 6V$

$v_c(\infty) = 10V$

$\tau = RC = 8\ sec$

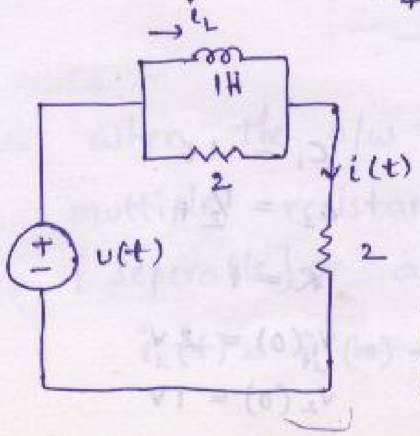
$v_c = 10 + (6 - 10)\, e^{-t/8}$

$= 10 - 4 \cdot e^{-t/8}, \quad t \geq 0$

$i_c = C \cdot \frac{dv_c}{dt} = e^{-t/8}$

$E_R = \int i^2 R\, dt$

$E_{4\Omega} = \int_0^\infty i^2(t) \cdot 4 \cdot dt = 16\,J$.



Determine $i(t)$, $t \geq 0$.

Assume $i_L(0) = 2A$.

(i). $i_L(0) = 2A$

(ii). $i_L(\infty) = \frac{1}{2} A$

(iii). $\tau = L/R_{eq} = 1\ sec$

$$i_L(t) = \tfrac{1}{2} + \left(2 - \tfrac{1}{2}\right) e^{-t/1}$$

$$= 0.5 + 1.5\, e^{-t}, \quad t \geq 0$$

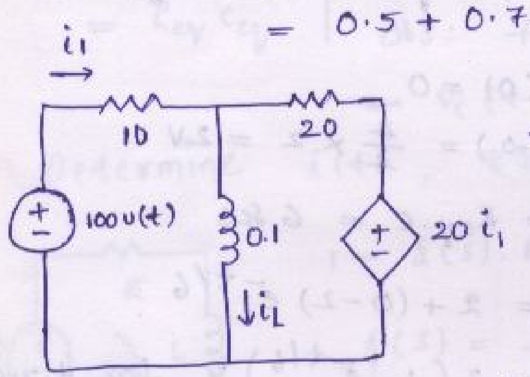
$$V_L(t) = L \cdot \frac{di_L(t)}{dt} = -1.5\, e^{-t}, \quad t \geq 0$$

$$V_L(t) = V_R(t) \Rightarrow V_L(t) = 2 \cdot i_R(t)$$

$$\Rightarrow i_R(t) = -0.75\, e^{-t}$$

$$i(t) = i_L(t) + i_R(t)$$

$$= 0.5 + 0.75\, e^{-t}, \quad t \geq 0$$



Determine i_1 & i_L for $t \geq 0$.

$$V_L = 0.1 \cdot \frac{di_L}{dt}$$

$$i_1 = \frac{100u(t) - V_L}{10} = 10u(t) - \frac{1}{100}\frac{di_L}{dt}$$

$$-i_1 + i_L + \frac{V_L - 20 i_1}{20} = 0$$

$$\Rightarrow \frac{1}{200}\frac{di_L}{dt} + i_L = 2 i_1$$

$$\frac{5}{200} \cdot \frac{di_L}{dt} + i_L = 20u(t)$$

$$i_L(0^-) = 0 = i_L(0^+)$$

$$\Rightarrow \frac{di_L}{dt} + 40 i_L = 800u(t)$$

$$s I_L(s) - i_L(0^+) + 40 I_L(s) = \frac{800}{s}$$

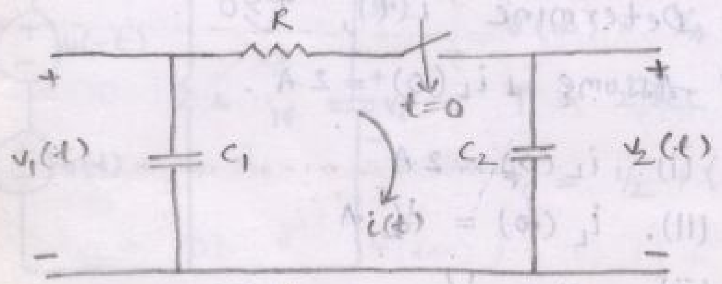
$$\Rightarrow I_L(s) = \frac{800}{s(s+40)} = \frac{20}{s} - \frac{20}{s+40}$$

$$i_L(t) = 20(1 - e^{-40t})\, u(t)$$

$$\tau = 1/40 \text{ sec}$$

$$i_1(t) = 10u(t) - \frac{1}{100}\left(20 \times 40 e^{-40t}\right) u(t)$$

$$= \left(10 - 8 e^{-40t}\right) u(t).$$



$C_1 = 1F$

$C_2 = \tfrac{1}{2} F$

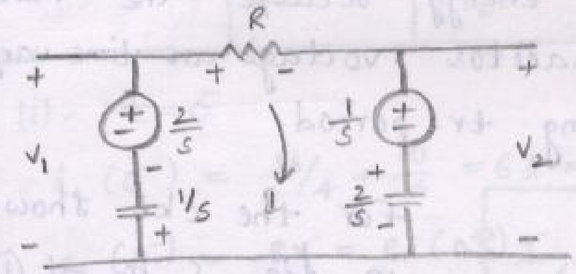
$R = 1$

$V_1(0) = 2V$

$V_2(0) = 1V$

Determine the SS voltages across the capacitors.

$V_1(\infty) = V_2(\infty) = (V_1 C_1 + V_2 C_2)/(C_1 + C_2) = \frac{5}{3}V$



$-\frac{1}{s}I(s) + \frac{2}{s} - 1\cdot I(s) - \frac{1}{s} - \frac{2}{s}I(s) = 0.$

$I(s) = \frac{1}{s+3}$

$i(t) = e^{-3t}, \; t \geq 0, \; \tau = 1/3$

$i(0^+) = 1$

$V_1(s) - \frac{2}{s} + \frac{1}{s} I(s) = 0$

$V_1(s) = \frac{2}{s} - \frac{1}{s}\cdot\frac{1}{s+3} = \frac{5/3}{s} + \frac{1/3}{s+3}$

$V_1(t) = \frac{5}{3} + 1/3 \cdot e^{-3t}, \quad t \geq 0$

$V_1(\infty) = 5/3\,V$

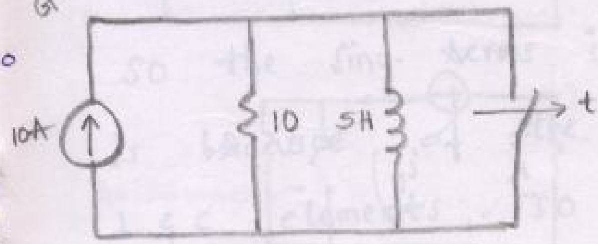
$V_2(s) - \frac{1}{s} - \frac{2}{s}\cdot I(s) = 0$

$V_2(s) = \frac{1}{s} + \frac{2}{s}\cdot\frac{1}{s+3} = \frac{5/3}{s} - \frac{2/3}{s+3}$

$V_2(t) = \frac{5}{3} - 2/3 \cdot e^{-3t}, \quad t \geq 0$

$V_2(\infty) = 5/3\,V$

G'02

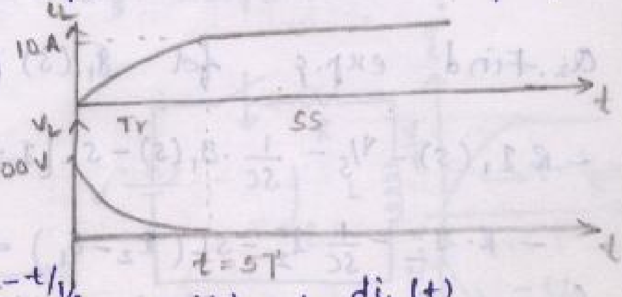


Determine the inductor current for $t \geq 0$. What is the energy stored by an inductor after a long time after the switch is opened.

(i). $i_L(0^-) = 0 = i_L(0^+)$

$i_L(\infty) = 10\,A$

$\tau = L/R = 1/2$ sec



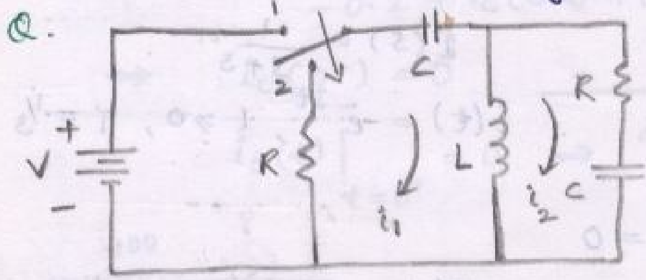
$i_L(t) = 10 + (0 - 10)\, e^{-t/1/2}, \quad V_L(t) = L\cdot\frac{di_L(t)}{dt}$

$= 10 - 10\,e^{-2t}, \; t \geq 0 \quad = 100\,e^{-2t}, \; t \geq 0$

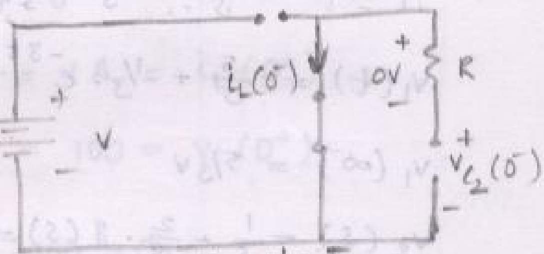
$$E_L\Big|_{t=5\tau \text{ or } t=\infty} = \frac{1}{2}\times 5\times 10^2 = 250\ J$$

NOTE :- Even though the excitation is DC, the L, C will store the energy because the inductor current and the capacitor voltage are time varying with +ve slope during tr. period.

Q. For the ckt shown in fig. S is at ① for a long time and moved to ② at t=0.



Q_1. Determine $i_1(0^+)$



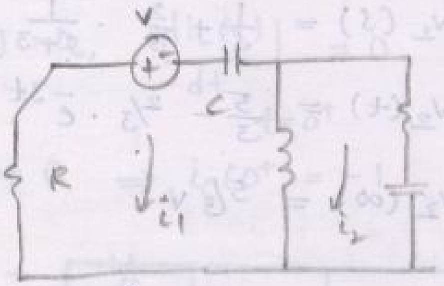
(i). $t=0^-$,

$i_L(0^-) = 0 = i_L(0^+)$

$v_{c_1}(0^-) = V = v_1(0^+)$

$v_{c_2}(0^-) = 0 = v_{c_2}(0^+)$

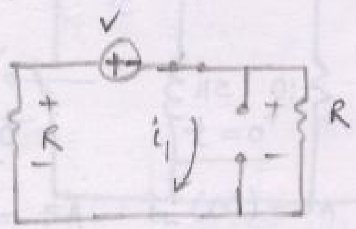
(ii). $t=0^+$,



$i_1(0^+) = i_2(0^+)$

$-V - i_1(0^+)[R+R] = 0$

$\Rightarrow i_1(0^+) = \frac{-V}{2R} = i_2(0^+)$



$i_L(t) = i_1(t) \sim i_2(t)$

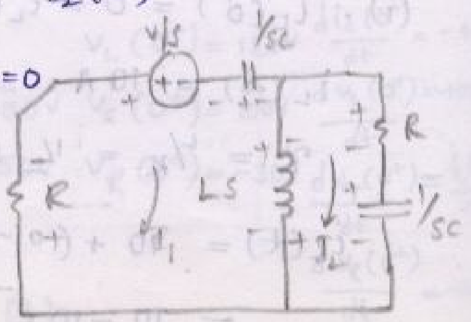
$\Rightarrow i_L(0^+) = 0$

$t=0^+$

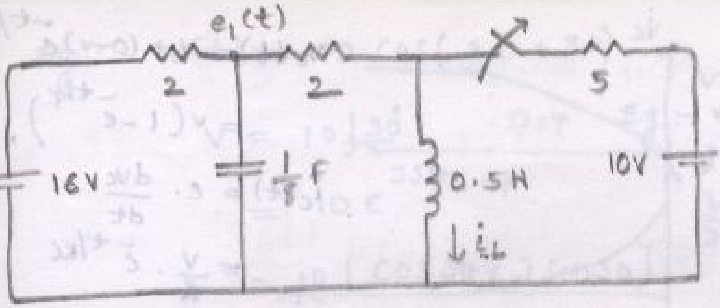
Q_2. Find exp.s for $I_1(s)$ and $I_2(s)$

$-R I_1(s) - V/s - \frac{1}{sC} I_1(s) - sL[I_1 - I_2] = 0$

$-R\cdot I_2 - \frac{1}{sC} I_2 - sL(I_2 - I_1) = 0$



$$\begin{bmatrix} R+sL+\frac{1}{sC} & -sL \\ -sL & R+sL+1/sC \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -V/s \\ 0 \end{bmatrix}$$

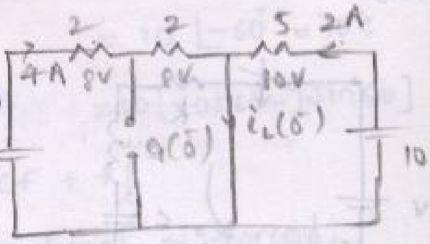


Determine $e_1(t)$ and $i_L(t)$ for $t > 0$.

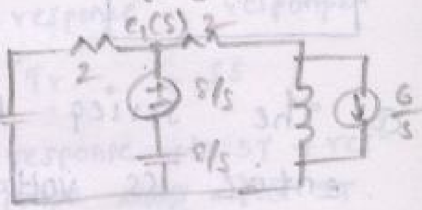
(i). $t = 0^-$

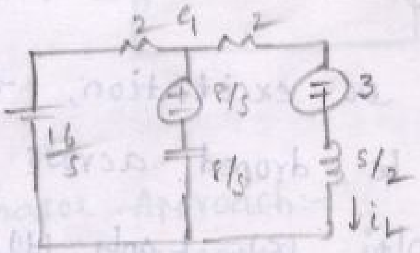
$i_L(0^-) = 16/4 + \frac{10}{5} = 6A = i_L(0^+)$

$e_1(0^-) = 8V = e_1(0^+)$



$t = 0$



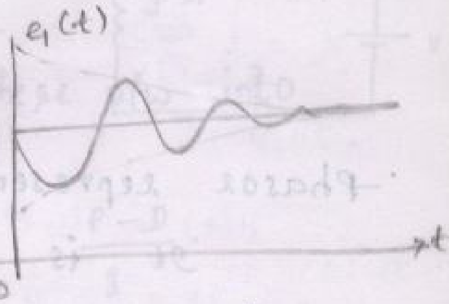


$$\frac{E_1(s) - 16/s}{2} + \frac{E_1(s) - 8/s}{8/s} + \frac{E_1(s) + 3}{2 + s/2} = 0$$

$$E_1(s) = \frac{8}{s}\left[\frac{s^2 + 6s + 32}{s^2 + 8s + 32}\right]$$

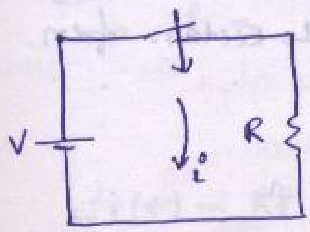
$$E_1(s) = \frac{8}{s}\left[1 - \frac{2s}{(s+4)^2 + 4^2}\right]$$

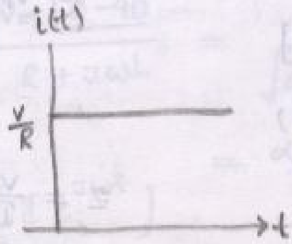
$=$

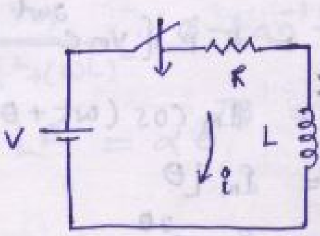


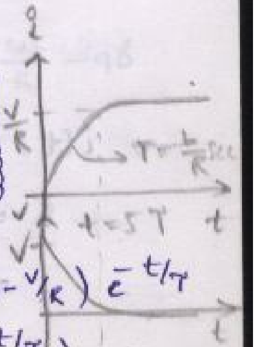
So the sinu. terms in the Tr. part of response is because of the exchange of energy b/w L & C elements, so called resonance.

Q. Determine the nature of response — ?



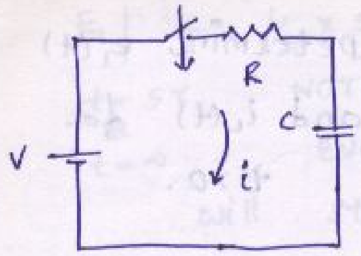


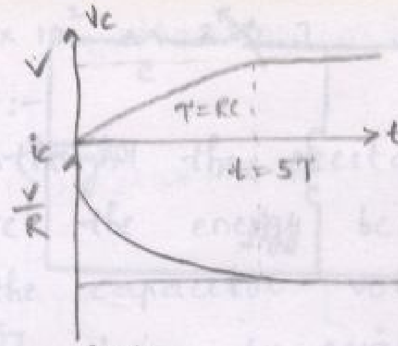




$i(t) = V/R + (0 - V/R)\,e^{-t/T}$

$i(t) = \frac{V}{R}(1 - e^{-t/T}), \; t > 0$



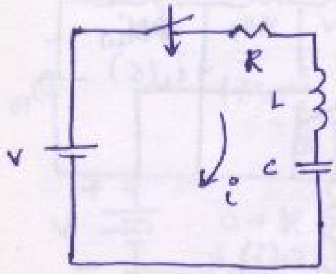


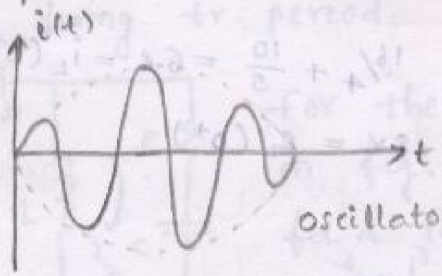
$$v_c(t) = V + (0 - V)e^{-t/\tau}$$

$$= V(1 - e^{-t/\tau}),\ t \geq 0$$

$$i_c(t) = C \cdot \frac{dv_c}{dt}$$

$$= \frac{V}{R} \cdot e^{-t/RC}$$

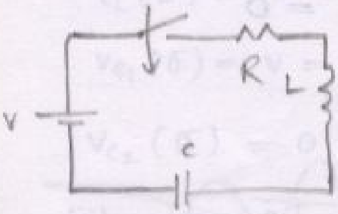


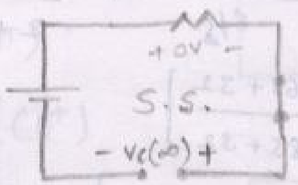


oscillatory response.

Q. The series RLC, with DC excitation, the entire SS voltage will be droped across

(a) R only (b) R & C only (c). L only (d) C only





$V = v_c(\infty)$

Ans: (d)

Phasor representation:-

It is defined only for the cosinu. signal

All sinu. signals are converted into cosinu. by subtracting 90° from the phase.

$\rightarrow$ $v(t) = V_m \cos(\omega t + \phi)$

$$= \text{R.P.}[V_m e^{j(\omega t + \phi)}]$$

$$= \text{R.P.}[V_m e^{j\omega t} \cdot e^{j\phi}]$$

$$= \text{R.P.}[V_m e^{j\omega t}]$$

$V = V_m e^{j\phi}$ – exponential form

$= V_m \angle \phi$

$= V_m[\cos\phi + j\sin\phi]$

↳ Rectan. form.

$\rightarrow$ $i(t) = I_m \cos(\omega t + \theta)$

$$I = I_m \angle \theta$$

$$= I_m e^{j\theta}$$

$$= I_m[\cos\theta + j\sin\theta]$$

$Z = \frac{V}{I}\ \Omega$

$Y = \frac{I}{V}\ \mho$

Eg: $v(t) = 10\cos(2t+30)$

$V = 10\angle 30^\circ$

$= 10e^{j30}$

$= 10[\cos 30 + j\sin 30]$

Eg:- $v(t) = 10\sin(2t+30)$

$= 10\cos(2t+30-90)$

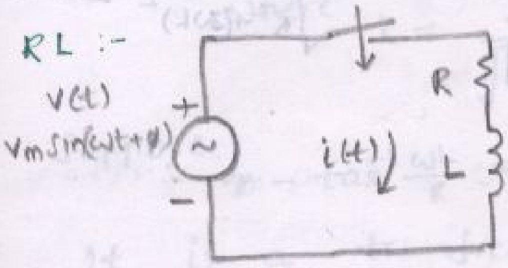
$= 10\cos(2t-60)$

$= 10\angle -60^\circ = 10e^{-j60}$

$= 10[\cos 60 - j\sin 60]$

AC Transients :-

RL :-



CS = CF + PI

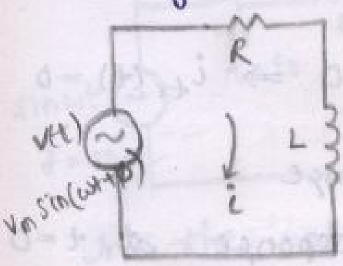
= natural response + forced response

= Tr. + SS

= response upto 5τ + response ~~upto~~ after 5τ.

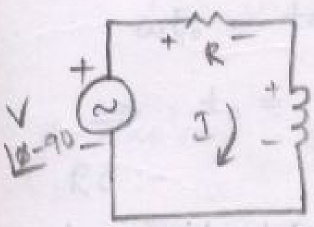
Phasor Approach:-

The responses in the n/w which is in SS with AC excitation are generally evaluated by using phasors.



Transform these n/w into phasor domain.

t − D	P − D
$i(t)$	I
$v(t)$	V
L	$j\omega L$
C	$\frac{1}{j\omega C}$
R	R



$V_m \angle\phi - 90 - RI - j\omega L \cdot I = 0$

$\Rightarrow I = \frac{V_m \angle \phi - 90^\circ}{R + j\omega L} = \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \angle \phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ$

$= \alpha \angle \beta = \alpha e^{j\beta}$

$i(t) = Re[I e^{j\omega t}]$

$= \alpha \cos(\omega t + \beta)$

$= \alpha \cos(\omega t + \phi - \tan^{-1}\frac{\omega L}{R} - 90^\circ)$

$$i_{ss}(t) = \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cdot \sin\left[\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right]$$

(2). LTA :-

$v(t) = V_m \sin(\omega t + \phi)$ $\qquad H(j\omega) = \frac{1}{R + j\omega L}$

$$= \frac{1}{\sqrt{R^2+(\omega L)^2}} \cdot \angle -\tan^{-1}\frac{\omega L}{R}$$

$$i_{ss}(t) = \frac{1}{\sqrt{R^2+(\omega L)^2}} \cdot V_m \sin\left[\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right]$$

$$i(t) = i_{tr}(t) + i_{ss}(t)$$

$$= k \cdot e^{-\frac{R}{L}t} + i_{ss}(t)$$

$$= k \cdot e^{-\frac{R}{L}t} + \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cdot \sin\left(\omega t + \phi - \tan^{-1}\frac{\omega L}{R}\right)$$

where,

$$k = \frac{-V_m}{\sqrt{R^2+(\omega L)^2}} \cdot \sin\left(\phi - \tan^{-1}\frac{\omega L}{R}\right) \ll 1$$

$$V_L = L \cdot \frac{di(t)}{dt}$$

Suppose $\phi - \tan^{-1}\frac{\omega L}{R} = 0 \Rightarrow k = 0 \Rightarrow i_{tr}(t) = 0$

$\therefore \; i(t) = i_{ss}(t) \rightarrow$ Tr. free response.

So the condi. for Tr. free response at $t=0$ is $\boxed{\phi = \tan^{-1}\frac{\omega L}{R}}$

ie $(\omega t + \phi)\Big|_{t=0} = \tan^{-1}\frac{\omega L}{R}$

NOTE : If the total ph. of excitation at the time of switching $= \tan^{-1}\frac{\omega L}{R}$ then no tr. r will result in the system at the time of switching for sinu. excitation.

$\rightarrow$ If the switch is closed at $t = t_0$ then the condi. for Tr. free response at $t = t_0$ is $\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R}$

→ If the excitation is $v(t) = V_m \cos(\omega t + \phi)$

then $i(t) = i_{tr} + i_{ss}$

$$= k \cdot e^{-\frac{R}{L}t} + i_{ss}$$

where $k = \frac{-V_m}{\sqrt{R^2 + (\omega L)^2}} \cos\left(\phi - \tan^{-1}\frac{\omega L}{R}\right)$

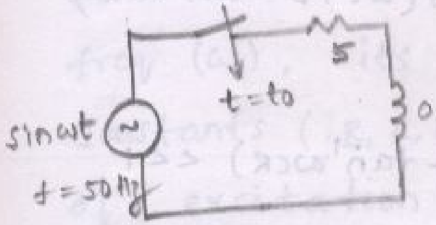
Suppose $\phi - \tan^{-1}\frac{\omega L}{R} = \pi/2 \Rightarrow k = 0$ & $i(t) = i_{ss}(t)$

It is a tr. free response

$\phi = \tan^{-1}\frac{\omega L}{R} + \pi/2$ at $t = 0$

$\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R} + \pi/2$ at $t = t_0$

G'09 The value of t_0, which results in tr. free response $t = t_0$ is — ?



$$\boxed{\omega t_0 = \tan^{-1}\frac{\omega L}{R}}$$

$$2\pi f\, t_0 = \tan^{-1}\frac{2\pi f L}{R}$$

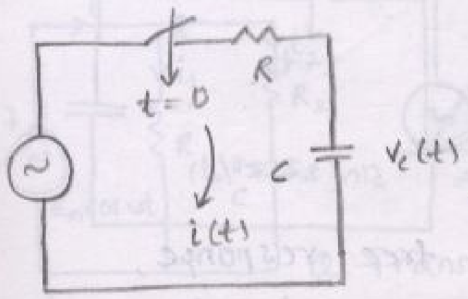
$$\Rightarrow t_0 = 1.78 \text{ ms}$$

In the above problem, if excitation is $\cos\omega t$.

$$\omega t_0 = \tan^{-1}\frac{\omega L}{R} + \pi/2$$

$$\Rightarrow t_0 = 6.78 \text{ ms.}$$

RC :-



$V_{c\,ss}(t)$ by LTA :-

$$\frac{V_c(s)}{V(s)} = H(s) = \frac{1}{1 + sCR}$$

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

$$= \frac{1}{\sqrt{1 + (\omega CR)^2}} \angle -\tan^{-1}\omega CR$$

$$v_c(t) = V_{ctr} + V_{css}$$

$$= k \cdot e^{-t/RC} + \frac{V_m}{\sqrt{1+(\omega CR)^2}} \sin(\omega t + \phi - \tan^{-1}\omega CR)$$

$$V_c(0^-) = 0 = V_c(0^+) = V_c(0)$$

where $k = \frac{-V_m}{\sqrt{1+(\omega CR)^2}} \sin(\phi - \tan^{-1}\omega CR) << 1$

$$i(t) = C \cdot \frac{dV_c(t)}{dt}$$

Suppose $\phi - \tan^{-1}\omega CR = 0 \Rightarrow k = 0 \Rightarrow V_c(t) = V_{css}(t)$

It is tr. free response.

$\phi = \tan^{-1}\omega CR$ at $t = 0$.

$\omega t_0 + \phi = \tan^{-1}\omega CR$, $t = t_0$

If the excitation is $v(t) = V_m \cos(\omega t + \phi)$

$$V_c(t) = V_{ctr}(t) + V_{css}(t)$$

$$= k \cdot e^{-t/RC} + \frac{V_m}{\sqrt{1+(\omega CR)^2}} \cos(\omega t + \phi - \tan^{-1}\omega CR)$$

$$V_c(0^-) = 0 = V_c(0^+) = V_c(0)$$

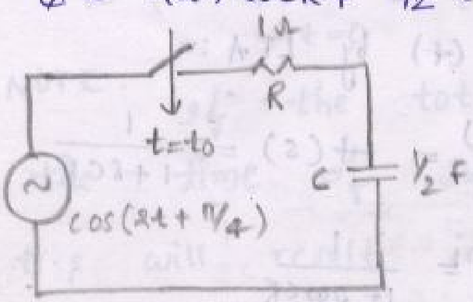
where $k = \frac{-V_m}{1+(\omega CR)^2} \cos(\phi - \tan^{-1}\omega CR) << 1$

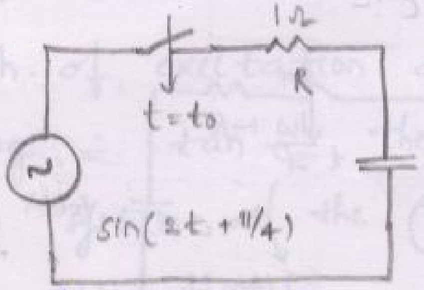
$i(t) = C\frac{dV_c(t)}{dt}$, Suppose $\phi - \tan^{-1}\omega CR = \pi/2$

$\Rightarrow$ $k = 0 \Rightarrow V_{ctr}(t) = 0 \rightarrow V_{ctr}(t) = V_{ss}(t)$

A tr. free response,

$\phi = \tan^{-1}\omega CR + \pi/2$ at $t = 0$ & $\omega t_0 + \phi = \tan^{-1}\omega CR + \pi/2$ at $t = t_0$.





The value of 't0' for the tr. free response,

$$2t_0 + \pi/4 = \tan^{-1}\omega CR + \pi/2$$

If in the above case excitation is $\sin(2t + \pi/4)$

∴ The value of $t_0 = 0$ sec. $2t_0 + \pi/4 = \tan^{-1}\omega CR$

The tr. free condi. is not possible for the n/w's with both energy storing elements ie for RLC. since the complex poles $s_1, s_2 = \alpha \pm j\beta$

$$i(t) = e^{\alpha t}(k_1 \cos\beta t + k_2 \sin\beta t) + i_{ss}(t)$$

Here k_1 and k_2 are functions of sin & cosine res. hence no time satisfieq k_1 and k_2 simultaneously zero so tr. term is always present.

$$\omega t_0 + \phi = \tan^{-1}\frac{\omega L}{R} + \pi/2$$

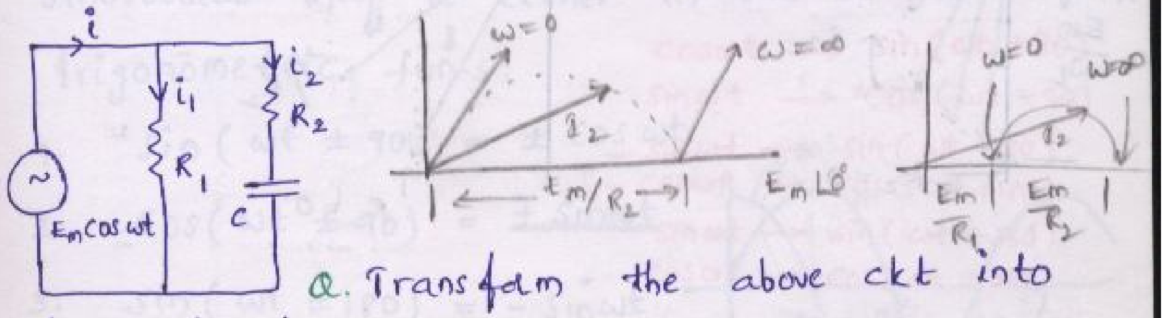
$$\omega t_0 + \phi = \tan^{-1}\omega CR + \pi/2$$

So, the tr. free time (t_0) in RL & RC ckts with AC excitation will depends on the source freq. (ω), its initial ph. (ϕ), on the ckt constants (R, L, C values) and on the nature of excitation [sin or cosine], but not on the max. value of the excitation. [V_m]

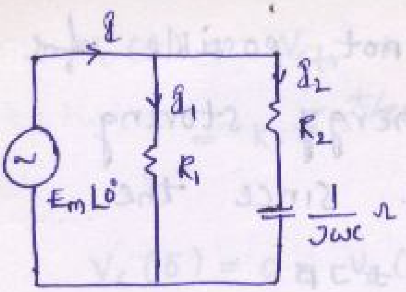
Locus Diagrams :-

G'01 EC

Q. In the ckt shown, the freq. ω is varied from 0 to ∞, the locus of the i phasor I_2 is -



Q. Transform the above ckt into Phasor domain



$$I = I_1 + I_2$$

$$I_1 = \frac{E_m\angle 0^\circ}{R_1}, \quad I_2 = \frac{E_m\angle 0^\circ}{R_2 + \frac{1}{j\omega C}}$$

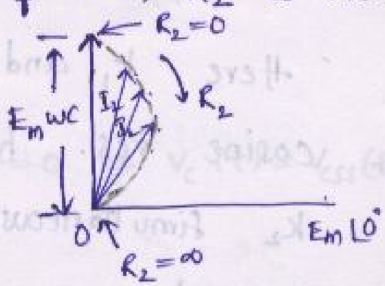
$\omega = 0 \Rightarrow I_2 = 0$

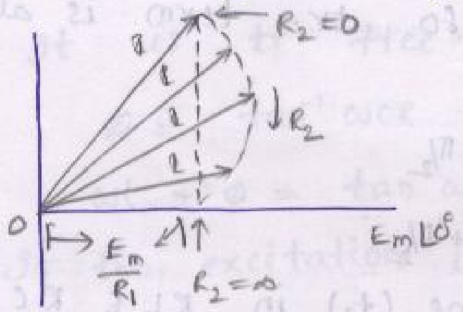
$\omega = \infty \Rightarrow I_2 = \frac{E_m\angle 0^\circ}{R_2}$

Q. In above problem, instead of ω, R_2 is varied to ∞,

$R_2 = 0, \quad I_2 = E_m \omega C \angle 90^\circ$

$R_2 = \infty \Rightarrow I_2 = 0$





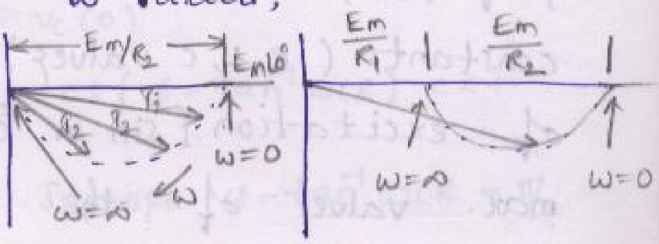
Q. In the above problem, instead of C, if inductive is used

$$I = I_1 + I_2$$

$$I_1 = \frac{E_m\angle 0^\circ}{R_1}$$

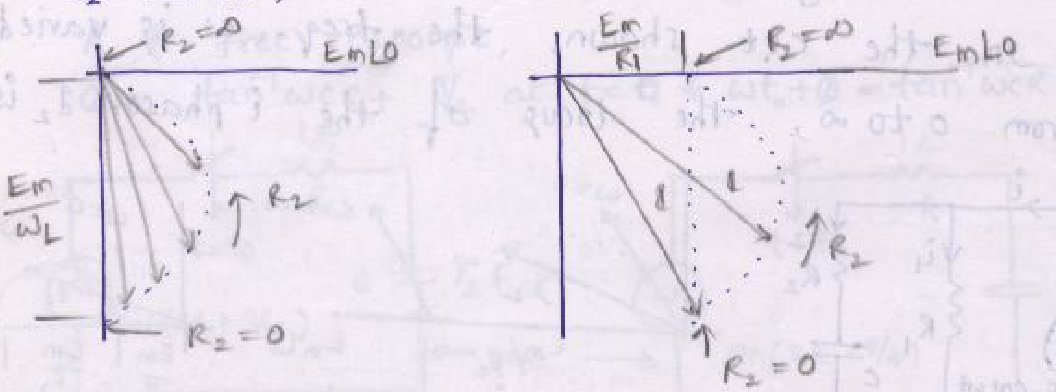
$$I_2 = \frac{E_m\angle 0}{R_2 + j\omega L}$$

ω varied,



R_2 varied,

4



AC fundamentals

$\int(\sin u) = \sin u$

$d(\sin u) = \sin u$

$\omega = \frac{\theta}{t} = \frac{360}{T} = \frac{2\pi}{T}$

$v(t) = V_m \sin\theta = V_m \sin\omega t = V_m \sin(\frac{2\pi}{T})t$

frequency = No. of cycles per second.

T sec — 1 cycle

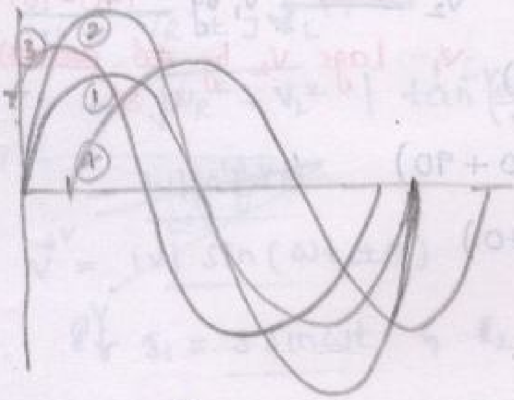
1 sec — ?

$\frac{1\ sec}{T\ sec} \times 1\ cycle$ = No. of cycle in one sec

$f = \frac{1}{T}$, $\omega = 2\pi \times \frac{1}{T} = 2\pi f$

$\Rightarrow v(t) = V_m \sin\omega t = V_m \sin(2\pi f)t$

Lead, Lag and inphase quantities :-



$v_1 = V_{m_1} \sin\omega t$

$v_2 = V_{m_2} \sin\omega t$

lead $v_3 = V_{m_3} \sin(\omega t + 30)$

$v_4 = V_{m_4}(\sin(\omega t - 45°)$ (lag)

if v_3 ref,

$v_3 = V_{m_3} \sin\omega t$

$v_1 = V_{m_1} \sin(\omega t - 30)$

$v_2 = V_{m_2} \sin(\omega t - 30)$

$v_4 = V_{m_4} \sin(\omega t - 75°)$

1. ph. difference can be obtained for the sinusoidals with same frequency.
2. while defining the ph. difference, all the sinusoidals may be either in sine or cosine form.

Trigonometric fun.s:

1. $\sin(\omega t \pm 90) = \pm \cos\omega t$
2. $\cos(\omega t \pm 90) = \underline{\mp \sin\omega t}$
3. $\sin(\omega t \pm 180) = -\sin\omega t$
4. $\cos(\omega t \pm 180) = -\cos\omega t$

$\cos\omega t \rightarrow \sin(\omega t + 90)$

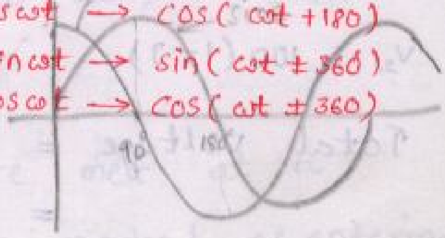
$\sin\omega t \rightarrow \cos(\omega t - 90)$

$-\sin\omega t \rightarrow \sin(\omega t + 180)$

$-\cos\omega t \rightarrow \cos(\omega t + 180)$

$\sin\omega t \rightarrow \sin(\omega t \pm 360)$

$\cos\omega t \rightarrow \cos(\omega t \pm 360)$



Q. $i_1 = 5 \sin \omega t$, $i_2 = 6 \cos \omega t$ then i_2 $\underline{\text{leads}}$ i_1 by $\underline{90^\circ}$

$i_2 = 6 \sin(\omega t + 90^\circ)$

Q. $i_1 = 5 \sin \omega t$, $i_2 = -6 \cos \omega t$ then i_2 $\underline{\text{lags}}$ i_1 by $\underline{90^\circ}$

$i_2 = 6 \cos(\omega t + 180^\circ)$

$= 6 \cos(\omega t + 180) = 6 \sin(\omega t + 180 + 90^\circ)$

$= 6 \sin(\omega t + 270)$

$= 6 \sin(\omega t + 360 - 90^\circ)$

$= 6 \sin(\omega t - 90^\circ)$

Q. $v_1 = -10 \cos(\omega t + 50^\circ)$,

$v_2 = 12 \sin(\omega t - 10^\circ)$. v_2 $\underline{\text{leads}}$ v_1 by $\underline{(40^\circ - 10^\circ)} = 30^\circ$

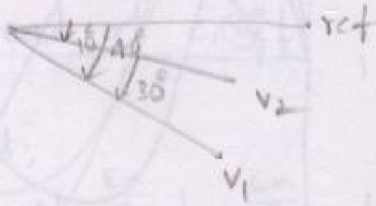
v_1 lags v_2 by ~~40°~~. ~~(20+10)~~ (40~10) $= 30^\circ$.

$v_1 = 10 \cos(\omega t + 50 + 180)$

$= 10 \sin(\omega t + 50 + 180 + 90)$

$= 10 \sin(\omega t + 360 - 40)$

$= 10 \sin(\omega t - 40)$



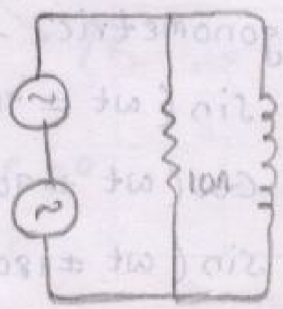
Q. $i_1 = 3 \sin(714t - 20^\circ)$

IES'97 $i_2 = -5 \cos(714t + 30^\circ)$ then i_2 $\underline{\text{lags}}$ i_1 by $\underline{40^\circ}$

Q. The ph. angle of current i w.r.t voltage of V_1 with the ckt shown is —

(a) 0° (b) $+45^\circ$ (c) -45° (d) -90°.

$V_1 = 100(1+j)$



$=$

$V_2 = 100(1-j)$

Total voltage $= V_1 + V_2$

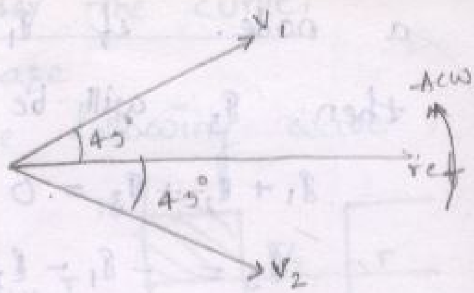
$= 200 + j0$

$I_R = \frac{V}{R} = \frac{200}{10} = 20\angle 0°$

$I_L = \frac{V}{X_L} = 20\angle -90°$

$I = I_R + I_L = 20\sqrt{2}\angle -45°$

I lags V_1 by $90°$.



AC Arithmetic Operations

representation of ac quantities

1. Instantaneous form $\rightarrow v(t) = V_m \sin(\omega t \pm \theta)$
2. polar form $\rightarrow V = V_m\angle \pm\theta$
3. Rectangular form $\rightarrow V = V_m(\cos\theta \pm j\sin\theta)$

$V = V_R \pm j\cancel{X}_L$

$= \sqrt{V_R^2 + V_L^2}\ \angle \tan^{-1}\left(\frac{V_L}{V_R}\right)$

$= |V| \angle \pm\theta$

$V = |V| \sin(\omega t \pm \theta)$

Q. If $i_1 = 3\sin\omega t$ & $i_2 = 4\cos\omega t$, then $i_1 + i_2 = ?$

$i_1 = 3\sin\omega t = 3\angle 0° = 3 + j0$

$i_2 = 4\cos\omega t = 4\sin(\omega t + 90)$

$= 4\angle 90°$

$\therefore i_1 + i_2 =$

Q. $i_1 = \sin\omega t$, $i_2 = -\cos\omega t$, then $i_1 + i_2 = -?$

$i_1 = 1\angle 0°$, $i_2 = \cos(180 + \omega t)$

$= \sin(180 + 90 + \omega t)$

$=$

Q. currents i_1, i_2 and i_3 are met at a junction. All currents are marked as entering a node. if $i_1 = -6\sin\omega t$, $i_2 = 8\cos\omega t$, then i_3 will be —.

Ans: $-10\cos(\omega t + 36.87^\circ)$

$$i_1 + i_2 + i_3 = 0$$

$$i_3 = -i_1 - i_2$$

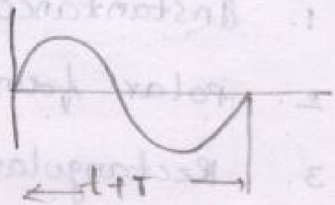
$$= 6\angle 0^\circ - 8\angle 90^\circ$$

$$=$$

Q. Average and RMS value :-

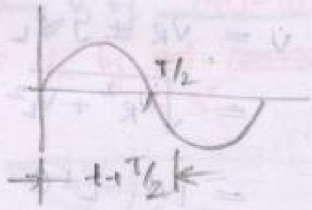
$f(t) = f(t+T)$

↳ periodic wave form.



$f(t) = f(t + T/2)$

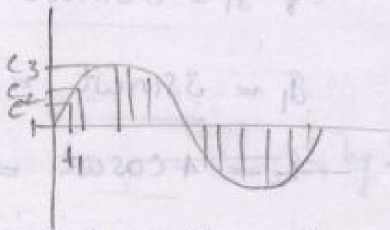
↳ Symmetric wave form.



- Instantaneous value
- Average
- rms value.

Average Value :-

$$\text{Avg. value} = \frac{e_1 + e_2 + \dots + e_n}{n}$$



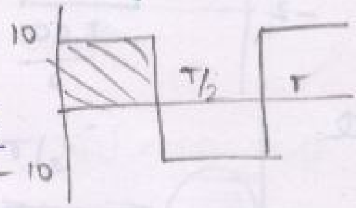
Rules:-

1. Generally avg. value of periodic symmetric sine wave is always zero.
2. If avg. value is required, for periodic sym. wave, it should be estimated for only +ve half cycle.
3. However for periodic unsym. waves, the avg. value should be estimated over full cycle.

So avg. value is not the effective value. only rms value is effective value.

Avg. value = $\dfrac{\text{Area under the curve}}{\text{Base}}$

Q. find avg. value for the following wave forms.

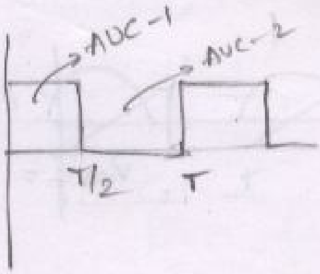


(1). Area under the curve $= 10V \times \frac{T}{2}$

Base $= T/2$

$V_{avg} = \dfrac{10V \times T/2}{T/2} = 10V$

(2).

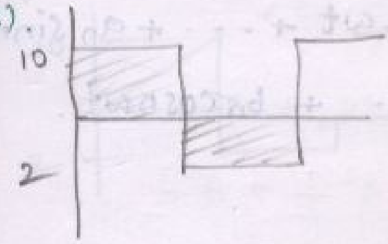


$AUC = AUC_1 + AUC_2$

$= 10 \times T/2 + 0$

Base $= T$.

(3).

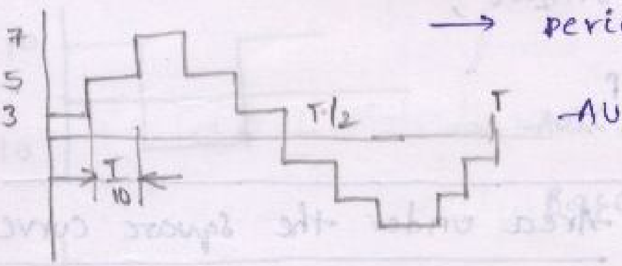


→ periodic unsym. curve.

so $AUC = (10 \times T/2) + (T/2 \times -2)$

Base $= T$.

(4).



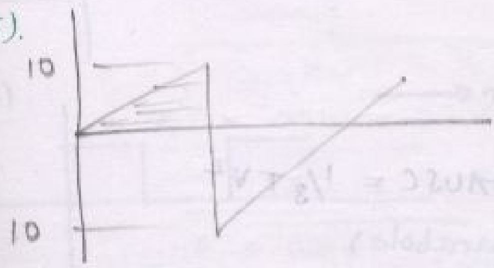
→ periodic symmetric.

$AUC = AUC_1 + AUC_2 + \cdots + AUC_5$.

$= 3 \times T/10 + 5 \times \frac{T}{10} + 7 \times \frac{T}{10} + 5 \times \frac{T}{10} + 3 \times \frac{T}{10}$

Base $= T/2$

(5).

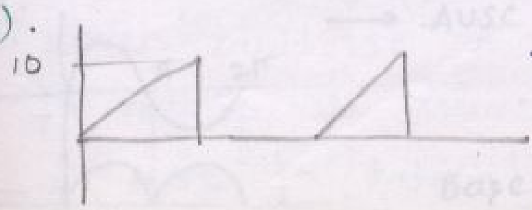


$AUC = \frac{1}{2} \times \text{base} \times \text{hiegh}$

$= \frac{1}{2} \times \frac{T}{2} \times 10$

Base $= T/2$

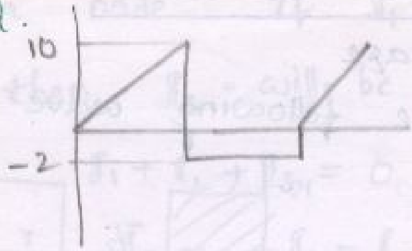
(6).



$AUC = AUC_1 + AUC_2$

$= \frac{1}{2} \times \frac{T}{2} \times 10 + 0$

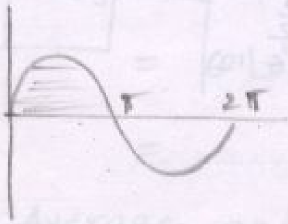
Base $= T$.

Q. 

$$AUC = AUC_1 + AUC_2$$

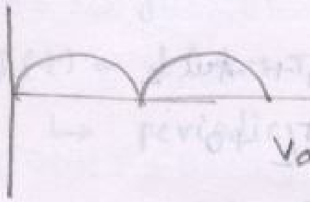
$$= \frac{1}{2} \times \frac{T}{2} \times 10 + \left(-2 \times \frac{T}{2}\right)$$

Base = T.

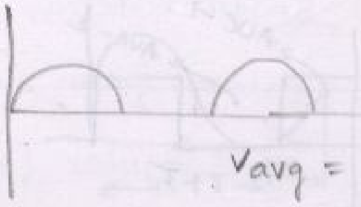
Q. 

$$AUC = \int_0^{\pi} V_m \sin\theta \cdot d\theta = +2V_m$$

Base = π rad ; $V_{avg} = \frac{2V_m}{\pi}$

Q. 

$V_{avg} = \frac{2V_m}{\pi}$



$V_{avg} = \frac{2V_m}{2\pi}$

Trigonometric Series :-

$$v(t) = a_0 + a_1 \sin\omega t + a_2 \sin 2\omega t + \cdots + a_n \sin n\omega t$$
$$+ b_1 \cos\omega t + b_2 \cos 2\omega t + \cdots + b_n \cos n\omega t.$$

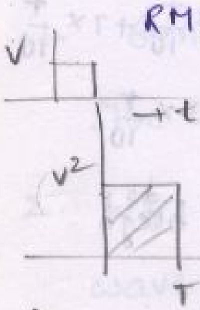
$\therefore V_{avg} = a_0.$

Ex: $i(t) = 5 + 2\sin 2\omega t$,

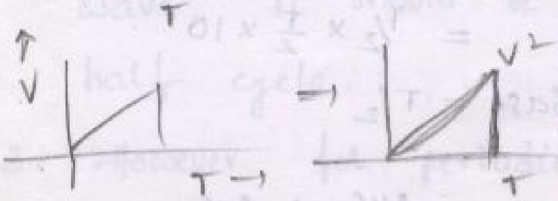
$I_{avg} = 5 \cdot$ Amp

RMS value :-

$$\text{RMS value} = \sqrt{\frac{\text{Area under the square curve}}{\text{Base}}}$$



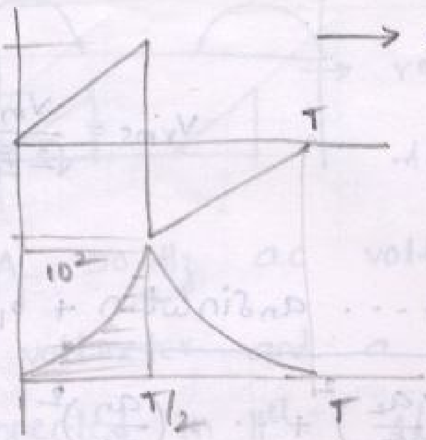
$AUSC = V^2 \times T$



$AUSC = \frac{1}{3} T V^2$ (parabola)

Q. find the rms values of following —

(1).



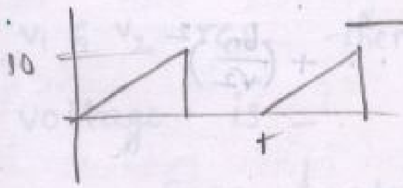
$\rightarrow$ $AUSC = \frac{1}{3} \times \frac{T}{2} \times 10^2$

$= \frac{100\,T}{6}$

$Base = T/2$

$V_{rms} = \sqrt{\frac{AUSC}{Base}} = 5.77\,V$

(2).

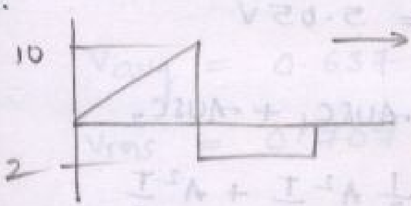


$\rightarrow$ $AUSC = AUSC_1 + AUSC_2$

$= \frac{1}{3} \times 100 \times \frac{T}{2} + 0$

$Base = T.$

(3).

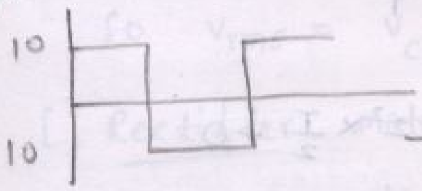


$\rightarrow$ $AUSC = AUSC_1 + AUSC_2$

$= \frac{1}{3} \times 10^2 \times \frac{T}{2} + 2^2\,T$

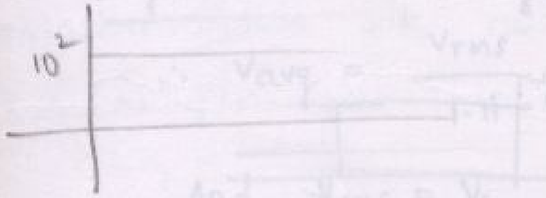
$Base = T.$

(4).

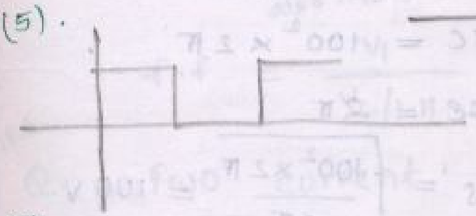


$\rightarrow$ $AUSC = 10^2\,T$

$Base = T.$



(5).

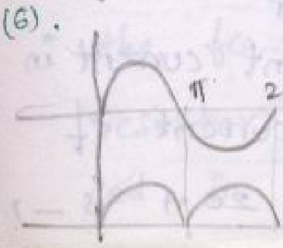


$\rightarrow$ $AUSC = AUSC_1 + AUSC_2$

$= 10^2 \times \frac{T}{2} + 0$

$Base = T.$

(6).



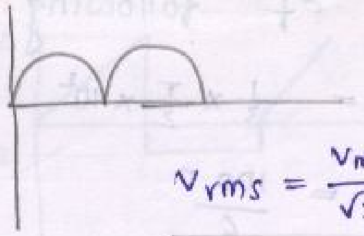
$\rightarrow$ $AUSC = \int_0^{\pi} V_m^2 \sin^2\theta \cdot d\theta$

$= \frac{V_m^2 \pi}{2}$

$Base = \pi.$

$V_{rms} = \frac{V_m}{\sqrt{2}}$

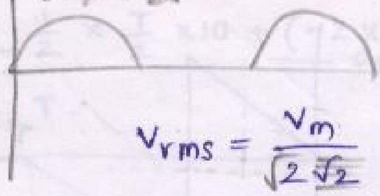
Q.



$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$AUSC = \frac{V_m^2 \pi}{2} + 0$$

Base = 2π



$$V_{rms} = \frac{V_m}{\sqrt{2}\sqrt{2}} = \frac{V_m}{2}$$

Trigonometric series :-

$$v(t) = a_0 + a_1 \sin \omega t + \cdots a_n \sin \omega t \cdot n + b_1 \cos \omega t + \cdots$$

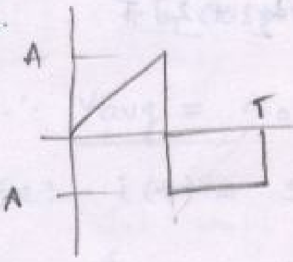
$$V_{rms} = \sqrt{a_0^2 + \left(\frac{a_1}{\sqrt{2}}\right)^2 + \left(\frac{a_2}{\sqrt{2}}\right)^2 + \cdots + \left(\frac{a_n}{\sqrt{2}}\right)^2 + \left(\frac{b_1}{\sqrt{2}}\right)^2 + \left(\frac{b_2}{\sqrt{2}}\right)^2 + \cdots + \left(\frac{b_n}{\sqrt{2}}\right)^2}$$

Ex: $v(t) = 5 + \cos 2\omega t$,

$$V_{rms} = \sqrt{5^2 + (1/\sqrt{2})^2} = 5.05\,V$$

G'95

Q.



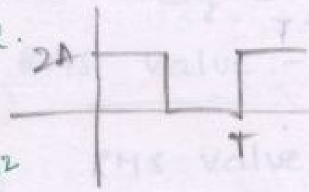
$\rightarrow$ AUSC = AUSC$_1$ + AUSC$_2$

$$= \frac{1}{3} A^2 \frac{T}{2} + A^2 \frac{T}{2}$$

Base = T

G'99

Q.

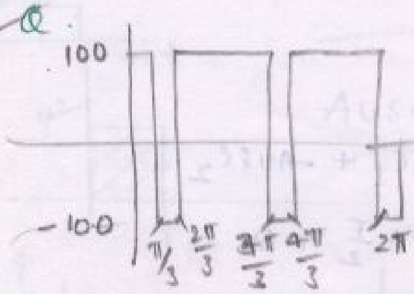


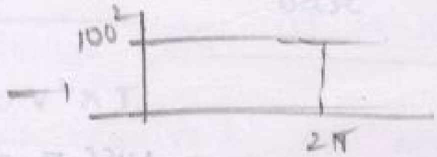
$\rightarrow$ AUSC = $2^2 \times \frac{T}{2} + 0$

Base = T.

G'02

Q.





$\rightarrow$ AUSC = $100^2 \times 2\pi$

Base = 2π

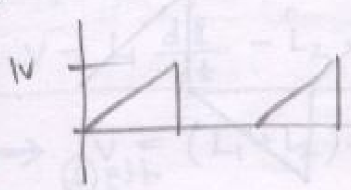
$$\rightarrow V_{rms} = \sqrt{\frac{100^2 \times 2\pi}{2\pi}} = 100\,V$$

G'04

Q The rms value of a resultant current in a wire, which carries dc current of 10 A, and sinu. AC current of 20 A is —?

$I_{rms} = \sqrt{10^2 + \left(\frac{20}{\sqrt{2}}\right)^2} = 17.32\ A.$

G'05

Q. 

→ rms value is — ?

Ans: $\frac{1}{6}$

IES'93

Q. A 50 Hz ac voltage is measured with HI voltmeter and a rectifier ac voltmeter connected in $\|^{el}$. If the meter readings are V_1 & V_2 res. then the form factor of AC voltage is —.

$$\text{form factor} = \frac{\text{rms value}}{\text{avg. value}}$$

$V_{avg} = 0.637\ V_m$

$V_{rms} = 0.707\ V_m$

$$f.f = \frac{V_m(0.707)}{V_m \cdot 0.637} = 1.11$$

Since HI meters reads true rms value.

so $V_{rms} = V_{calibrated}$.

[Rectifier meters are calibrated to read avg. value directly)

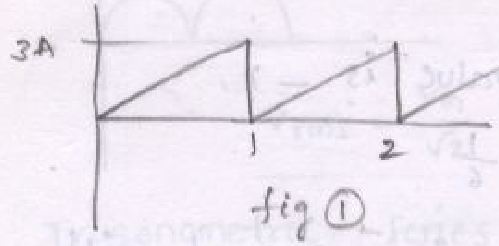
$$\therefore V_{avg} = \frac{V_{rms}}{1.11} = \frac{V_2}{1.11}$$

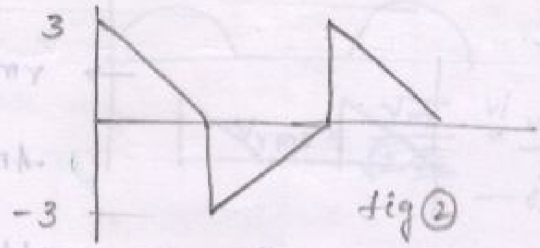
And also $V_{rms} = V_1$

$$f.f = \frac{V_1}{V_2/1.11}$$

IES

Q. Two current wave forms are shown in the fig. ① & ② are passed to identical resistors of 1Ω. The ratio of heat produced in each resistors in a given time by current of fig ① to fig ② is — ?





$i_{rms} = \sqrt{3}\ A$ $\qquad$ $i_{rms} = \sqrt{3}\ A$

NOTE :- The wave form which is having highest rms value will produce more heat.

Since the rms values of both wave forms are same, the ratio of heat produce is 1:1

Magnetically Coupled Circuits :-

→ The opposition of change of current is called inductance.

→ In T/f, $\phi_1 = \phi_{11} + \phi_{12}$, co. of coupling $k = \dfrac{\phi_{12}}{\phi_1}$

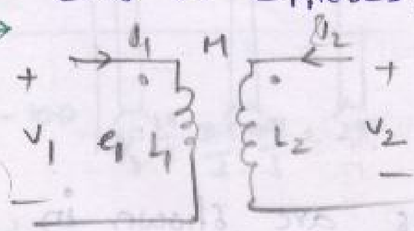
$$e_m = N_2 \cdot \frac{d\phi_{12}}{dt}$$

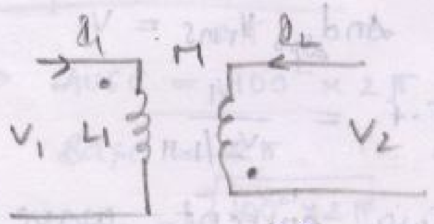
Dot convension:

→ To understand the winding sence.

→ To understand what ever the mutual inductance (mutual induced emf) aids the self induced emf & opposes.

→



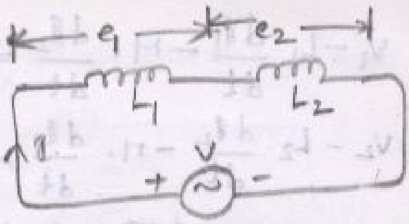


$V_1 - L_1 \dfrac{di_1}{dt} - M \cdot \dfrac{di_2}{dt} = 0$ $\qquad$ $V_1 - L_1 \dfrac{di_1}{dt} + M \cdot \dfrac{di_2}{dt} = 0$

$V_2 - L_2 \dfrac{di_2}{dt} - M \cdot \dfrac{di_1}{dt} = 0$ $\qquad$ $V_2 - L_2 \cdot \dfrac{di_2}{dt} + M \cdot \dfrac{di_1}{dt} = 0$

Series ckt :

a). without magnetic coupling :-

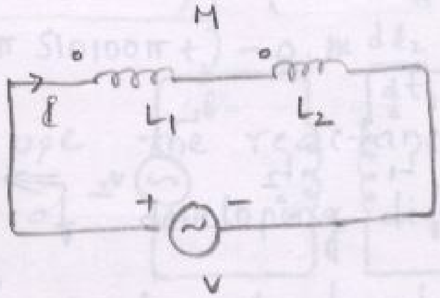


$$V - L_1 \frac{di}{dt} - L_2 \cdot \frac{di}{dt} = 0$$

$$\Rightarrow V = (L_1 + L_2) \cdot \frac{di}{dt}$$

$$\Rightarrow L_{eq} = L_1 + L_2 .$$

b). Series aiding :-

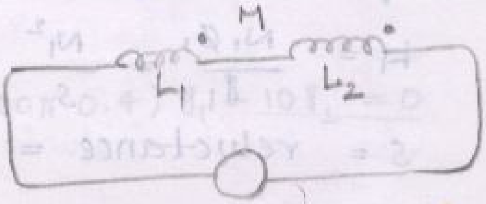


$$V - \left(L_1 \frac{di}{dt} + M \cdot \frac{di}{dt}\right)$$

$$- \left(L_2 \frac{di}{dt} + M \cdot \frac{di}{dt}\right) = 0$$

$$\Rightarrow V = (L_1 + L_2 + 2M) \frac{di}{dt} \Rightarrow \text{so } L_{eq} = L_1 + L_2 + 2M .$$

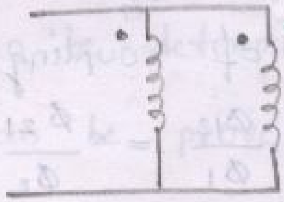
c). Series opposing :-



$$V - \left(L_1 \frac{di}{dt} - M \cdot \frac{di}{dt}\right)$$

$$- \left(L_2 \frac{di}{dt} - M \cdot \frac{di}{dt}\right) = 0$$

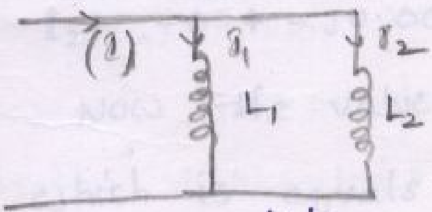
$$\Rightarrow L_{eq} = L_1 + L_2 - 2M .$$



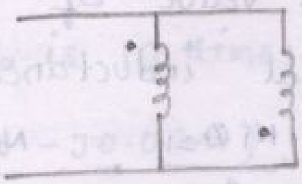
parallel aiding

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

parallel circuit :



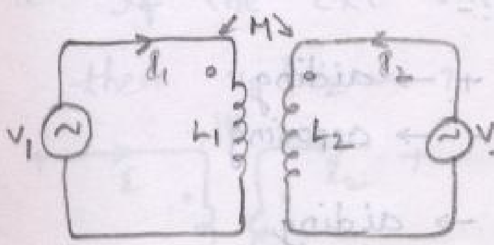
$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$$



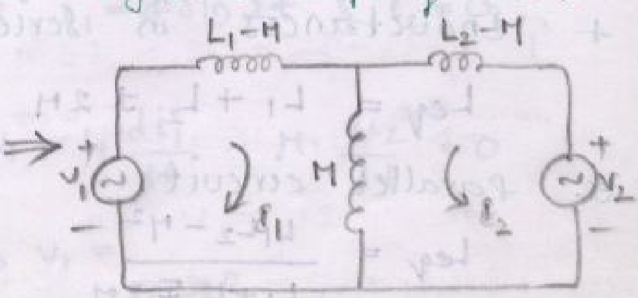
parallel opposing :-

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

Conductive and inductive magnetic coupling circuit :-



inductively coupled



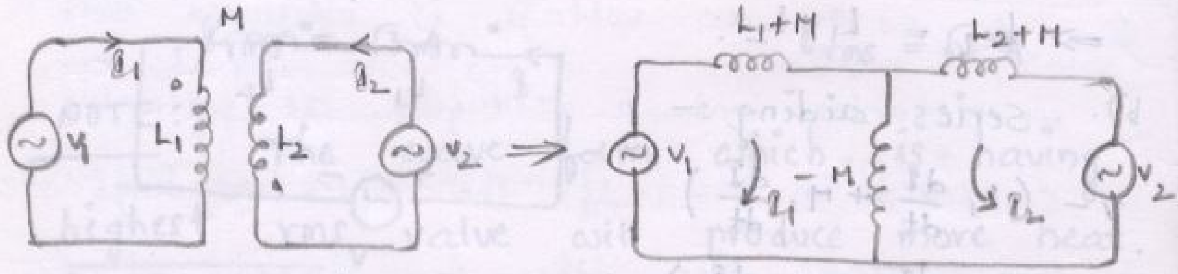
conductively coupled

$$V_1 - L_1\frac{di_1}{dt} - M\cdot\frac{di_2}{dt} = 0 \rightarrow (1) \qquad V_1 - (L_1 - M)\frac{di_1}{dt} - M\frac{d(i_1+i_2)}{dt} = 0 \rightarrow$$

$$V_2 - L_2\frac{di_2}{dt} - M\cdot\frac{di_1}{dt} = 0 \rightarrow (2) \qquad V_2 - (L_2 - M)\frac{di_2}{dt} - M\cdot\frac{d(i_1+i_2)}{dt} = 0$$

$\Rightarrow$ eq. (1) = eq. (3) & eq. (2) = eq. (4)

case II :



Summary of formulae :

1. Self inductance :

$$L_1 = \frac{N_1 \phi_1}{i_1} = \frac{N_1^2}{S}$$

$$S = \text{reluctance} = \frac{l}{\mu_0 \mu_r A}$$

$$L_2 = \frac{N_2 \phi_2}{i_2} = \frac{N_2^2}{S}$$

2. coe. of coupling (k) :

$$k = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2} \qquad \text{where } \phi_1 = \phi_{11} + \phi_{12}$$
$$\phi_2 = \phi_{21} + \phi_{22}$$

Max. value of k is 1.

3. Mutual inductance :-

$$M = \frac{N_1 \phi_{21}}{i_2} = \frac{N_2 \phi_{12}}{i_1}$$

and also $M = k\sqrt{L_1 L_2}$

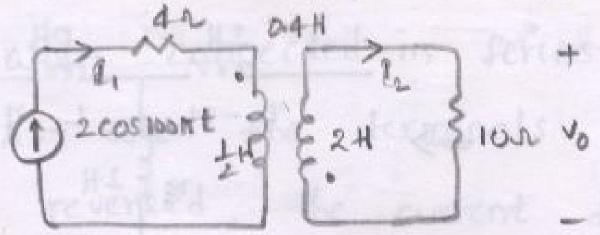
4. Inductances in series :-

$$L_{eq} = L_1 + L_2 \pm 2M \qquad \begin{matrix} + \rightarrow \text{aiding} \\ - \rightarrow \text{opposing} \end{matrix}$$

5. parallel circuit :-

$$L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M} \qquad \begin{matrix} - \rightarrow \text{aiding} \\ + \rightarrow \text{opposing} \end{matrix}$$

Q. The value of v_0 — ?



$-\frac{1}{2}\left(\frac{d}{dt}(2\cos 100\pi t)\right)$

$$v_0 = -4I_1 - \tfrac{1}{2}(-200\pi \sin 100\pi t) - 0.4\frac{dI_2}{dt}$$

It is convinient to use the reactance voltage drop instead of developing differential eq. If the value of I_2 is not found, then the value of v_0 can't determine.

from the II loop:

$$-j(100\pi \cdot 2) \cdot I_2 - j(100\pi\, 0.4)\, I_1 - 10 I_2 = 0$$

from the I loop:

$$-4I_1 - j(100\pi \times \tfrac{1}{2})\, I_1 - j(100\pi \times 0.4)\, I_2 = 0$$

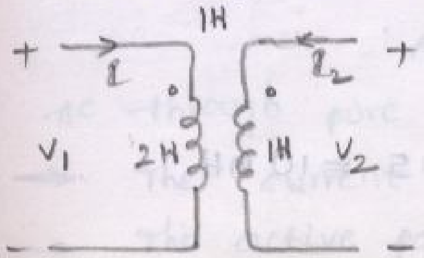
The value of I_2 can be determine using the above eq. [and can be proved as

$I_2 = -0.4 - j0.0064$.

Now the value of v_0 is 10 times of I_2 which is equals to $(-4 - j0.064)$.

so $v_0 = 4\cos 100\pi - 179.1$

Q. If the ckt shows $i_1 = 4\sin 2t$ & $i_2 = 0$, then v_1 & v_2 — ?

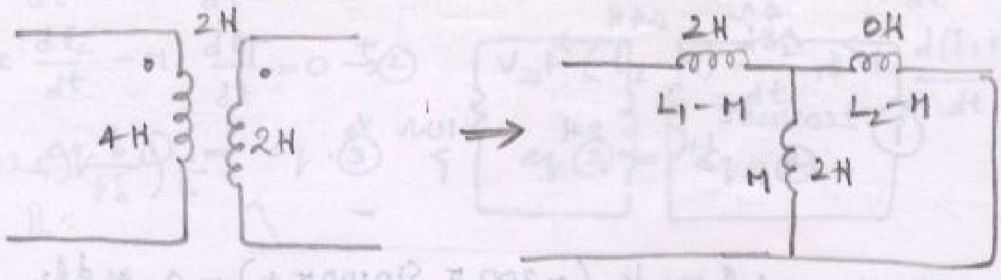


$$v_1 - L_1\frac{di_1}{dt} - M\cdot\frac{di_2}{dt}^{\to 0} = 0$$

$\Rightarrow v_1 =$

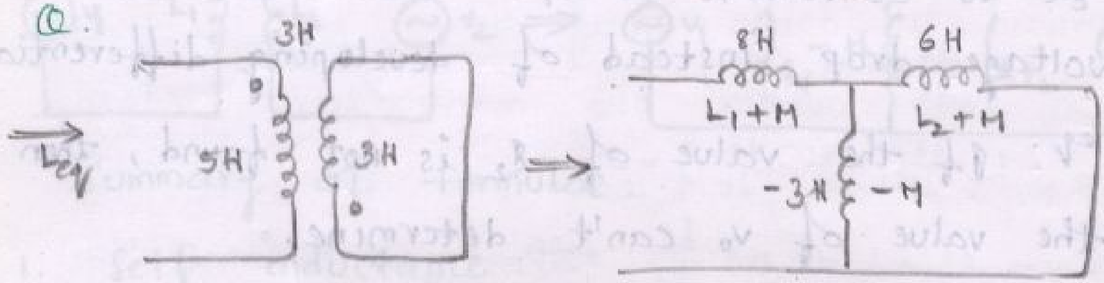
$$v_2 - L_2\frac{di_2}{dt}^{\to 0} - M\frac{di_1}{dt} = 0$$

Q. find L_{eq}.



$\Rightarrow L_{eq} = 2H$

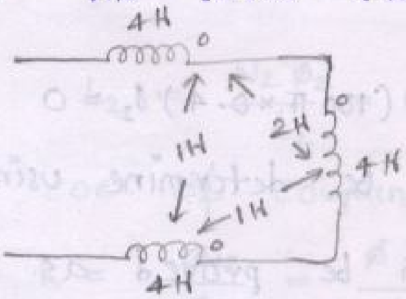
Q.



$$L_{eq} = 8 + \frac{6 \times (-3)}{6-3} = 2H.$$

G'92
IES'95

Q. The total inductance of the circuit is –.



$$L_{eq} = L_1 + L_2 + L_3 - 2M_{12} + 2M_{23} - 2M_{31}$$

$$= 8H$$

IES'95

Q. 2 coils having inductance $L_1 = 2mH$, $L_2 = 15mH$ have effective inductance of 40 mH, when connected in series aiding. What will be effective inductance if we connect them in series, opposition.

$$L_1 + L_2 + 2M = 40 \text{ mH}$$

$$\Rightarrow 10 + 15 + 2M = 40$$

$$\Rightarrow 2M = 15 \text{ mH}$$

$$\& \; L_1 + L_2 - 2M = 10 + 15 - 15 = 10 \text{ mH}$$

So Ans: -10 mH

Q. Two identical coils of negligible resistance when connected in series with a current of 10 A. when the terminals of one of the coil is reversed, the current drawn is 8 A, then the coe. of coupling — ?

$L_1 = L_2 = L$

$$I_1 = 10\,A = \frac{V}{J[\omega L_1 + \omega L_2 - 2\omega M]}$$

$$I_2 = 8\,A = \frac{V}{J[\omega L_1 + \omega L_2 + 2\omega M]}$$

$$\Rightarrow \frac{10}{8} = \frac{L_1 + L_2 + 2M}{L_1 + L_2 - 2M} = \frac{2L + 2M}{2L - 2M}$$

$$\Rightarrow \frac{L+M}{L-M} = \frac{10}{8} \Rightarrow L = 9M$$

Also $M = k\sqrt{L_1 L_2}$

$= k\sqrt{LL} = kL$

$$\Rightarrow k = \frac{M}{L} = \frac{1}{9}$$

Q. Two inductive coils L_1 & L_2 are magnetically coupled in series opposing & in parallel aiding res. The mutual inductance b/w the coil is M. The equivalent inductances in the two ~~coils~~ cases will be — ?

$$L_1 + L_2 - 2M \quad \& \quad \frac{L_1 L_2 - M}{L_1 + L_2 - 2M}$$

AC circuits

AC through pure resistance :-

→ The current $i(t)$ in phase with voltage $v(t)$

→ The active power 100%, reactive power 0%

→ power is double freq. transient. (freq. of power = 100 Hz) → power is unidirectional.

AC through pure inductance:-

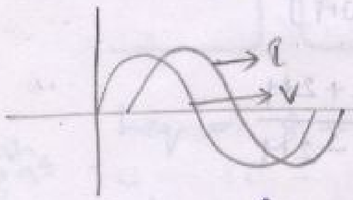
$$V_L(t) = L \cdot \frac{di(t)}{dt} = L \cdot \frac{d}{dt}[I_m \sin\omega t]$$

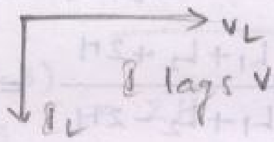
$$= I_m L \cdot \omega \cos\omega t$$

$$= V_m \cdot \sin(\omega t + 90), \text{ where } V_m = I_m \omega L.$$

Inductive reactance $X_L = \omega L = 2\pi f L$

→ voltage leads the current by 90°.





I lags V by 90°

→ power is double freq. transient [∵ f of V & f of I.

→ power is bidirectional.

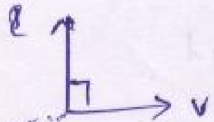
→ active power = 0% [There is no any power utilised ~~by~~ but it is transferred from load to source, source to load alternatively.

Reactive power Q = 100%

AC through pure capacitor:-

$$i(t) = C \cdot \frac{dV(t)}{dt}$$

4 $$= \frac{V_m}{X_C} \cdot \sin(\omega t + 90), \quad X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

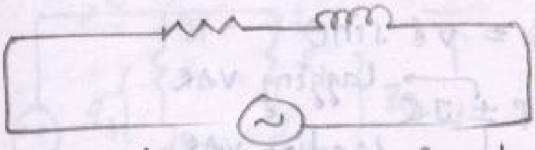


I leads V by 90°.

NOTE:- remain 3 points are similar to inductance.

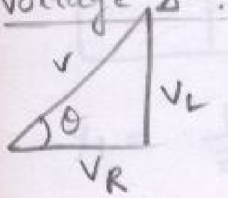
AC Series Circuits

R-L Series circuit :-

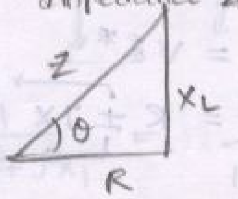


$\bar{V} = \bar{V}_R + \bar{V}_L$

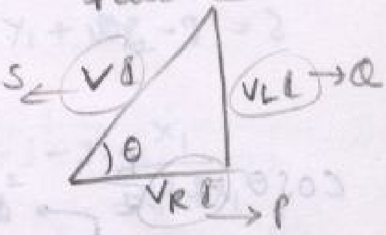
voltage Δ^{le} :-



Impedance Δ^{le}



Power Δ^{le}



$V = V_R + jV_L$

$|V| = \sqrt{V_R^2 + V_L^2}$

$|Z| = \sqrt{R^2 + X_L^2}$

$Z = R + jX_L$

P = active power $= V_R I = VI\cos\theta$ (watt)

S = Apparent or True or total power (VA)

Q = Reactive power $= V_L I = VI\sin\theta$ (VAR)

from power Δ^{le},

complex power $S = P + jQ$

$P = VI\cos\theta = S\cos\theta$

$Q = VI\sin\theta = S\sin\theta$

$\cos\theta \rightarrow$ pf. , θ angle b/w Supply voltage and supply current.

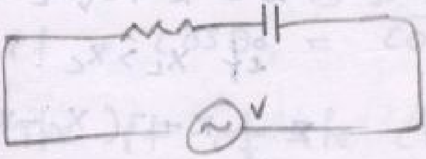
$\cos\theta = pf = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$.

Generally voltage is taken as reference.

$I = |I| \angle -\theta$ & $V = |V| \angle 0^\circ$
lags 'V'.

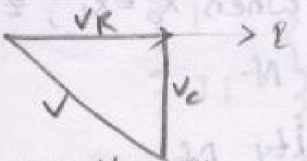
$S = VI^* = V\angle 0^\circ \cdot I \angle +\theta$
[As convensional]

RC Series Circuit :-



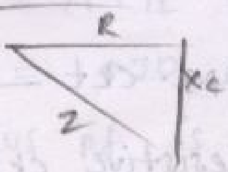
$\bar{V} = \bar{V}_R + \bar{V}_C$

voltage Δ^{le} :-



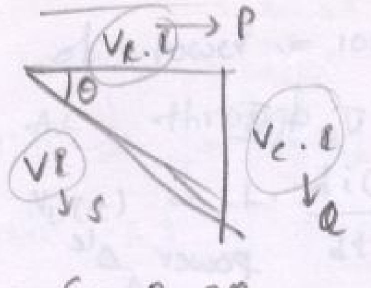
$V = V_R - jV_C$

Impedance Δ^{le} :-



$Z = R - jX_C$

power Δ^{le} :-



power factor $= \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$

$P = VI\cos\theta$

$Q = VI\sin\theta$

$S = P \pm jQ$ → lagging VAR (+), leading VAR (−)

$S = P - jQ$

$S = VI^*$

$Z = R \pm jX$ → inductive (+), capacitive (−)

$\cos\theta = \frac{R}{Z}$

$Z = |Z| \underline{\left| \pm \tan^{-1}\left(\frac{X}{R}\right)\right.}$ → indu. (+), capacitive (−)

$Q = +ve$, indu. VAR

$= -ve$, capa. VAR

Eg :- $Z = 10 \underline{|-360}$ ⟶ RC ckt

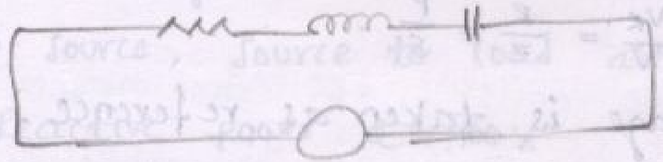
$Z = 12 \underline{|72}$ ⟶ RL ckt

$Z = 10 \underline{|0^\circ}$ ⟶ pure resistor

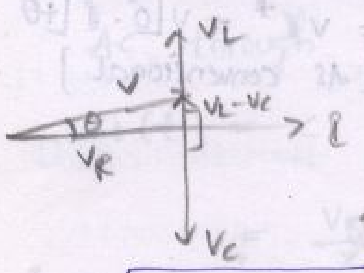
$Z = 10 \underline{|90^\circ}$ ⟶ pure inductor

$Z = 10 \underline{|-90^\circ}$ ⟶ pure capacitor.

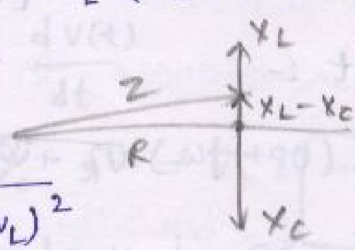
R-L-C series circuit :-



$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$.



V_L & V_C are 180° out of phase



$Z = R + j(X_L - X_C)$ if $X_L > X_C$.

$Z = R + j(X_C - X_L)$ if $X_C > X_L$.

when $X_C = X_L$, $Z = R$.

$V = \sqrt{V_R^2 + (V_C \mp V_L)^2}$

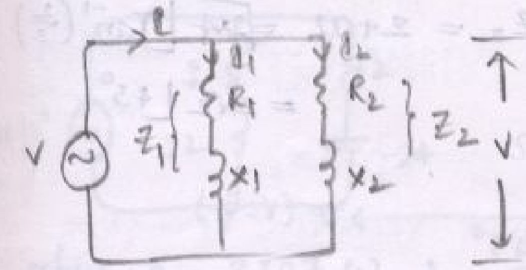
when $X_L > X_C$ → Indu. ckt → lagging pf

$X_L < X_C$ → capa. ckt ⟶ leading pf

$X_L = X_C$ → Resistive ckt ⟶ unity pf.

power factor $= \cos\theta = \frac{R}{Z} = \frac{R}{\sqrt{R^2+(X_L-X_C)^2}}$

parallel circuit :-



$I_1 = \frac{V}{Z_1}$; $I_2 = \frac{V}{Z_2}$

$I = I_1 + I_2$

$I = VY_1 + VY_2$

$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + jX_1} = \frac{R_1}{R_1^2+X_1^2} - j \cdot \frac{X_1}{R_1^2+X_1^2}$

$\Rightarrow Y_1 = G_1 - jB_1$ (G_1 → conductance, B_1 → susceptance)

$Y_2 = G_2 - jB_2$

$Z = R \pm jX$ (+ → L, − → C)

$Y = G \mp jB$ (− → L, + → C)

$Z = |Z| \angle \pm\theta$ (+ → lag pf, − → lead pf)

$Y = |Y| \angle \pm\theta$ (+ → lead pf, − → lag pf.)

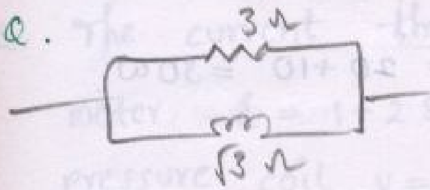
$|Y| = \sqrt{G^2+B^2}$

$\theta = \tan^{-1}\left(\frac{B}{G}\right)$

$Y_{total} = Y_1 + Y_2$

power factor of the circuit :-

Q. The pf of the circuit — ?

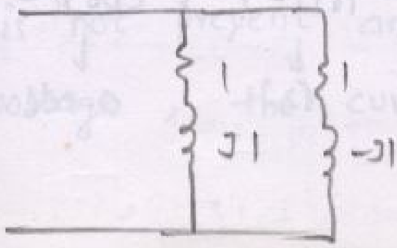


$Y_{total} = Y_1 + Y_2 = \frac{1}{3} - j\frac{1}{\sqrt{3}}$

$\theta = \tan^{-1}\left(\frac{1/\sqrt{3}}{1/3}\right) = 60^\circ$

$pf = \cos\theta = \cos 60^\circ = 0.5$ lag.

Q. The pf of the circuit — ?

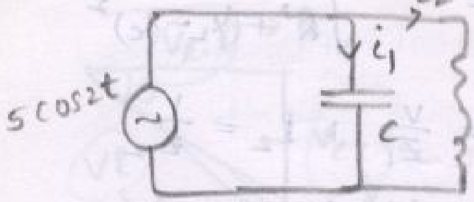


$Y = Y_1 + Y_2$

$= 1 + j0$

Thus pf is unity.

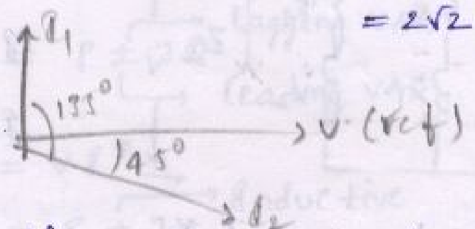
Q. for this ckt, i_1 leads i_2 by — angle.



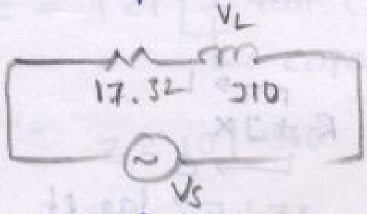
$\omega = 2$

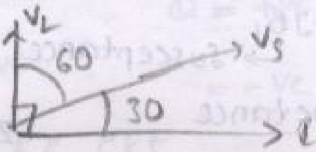
$Z_2 = 2 + j2 = \sqrt{2\times4} \angle \tan^{-1}\left(\frac{2}{2}\right)$

$= 2\sqrt{2} \angle 45^\circ$



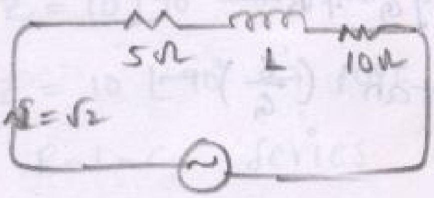
Q. for the ckt, the voltage V_L has a ph. angle of — w.r.t. V_s.





V_L leads V_s by 60°.

Q. In the ckt shown the power consumed by 5Ω is 10W. then the pf of the ckt is—?



$V = 50\cos\omega t$

$P_{5\Omega} = 10W$

I = rms current.

$I^2 \times 5 = 10 \rightarrow I = \sqrt{2}A$

$P_{10\Omega} = I^2 \times 10 = 20W$

Total active power $= P = 20 + 10 = 30W$

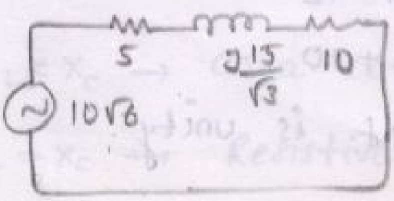
$S = VI$

$= \frac{50}{\sqrt{2}} \cdot \sqrt{2} = 50\ VA$

$\cos\theta = \frac{P}{S} = \frac{30}{50} = 0.6$ lag.

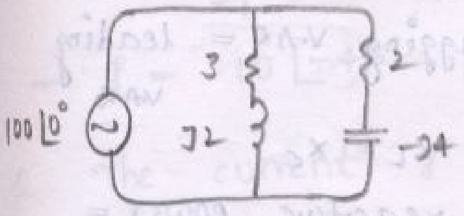
Q. Consider the following ckt, if the power consumed by $5\Omega = 10W$ then $|I| = ?$ & $\cos\phi = ?$

$\downarrow$ $\sqrt{2}$ $\downarrow$ 0.866



Model-6 : power calculations :-

Q. The complex power drawn by the circuit is -?



$Y_1 = \frac{1}{3+J2}$; $Y_2 = \frac{1}{2-J4}$

$S = V I^*$

$= V^2 (Y_1^* + Y_2^*)$

$= 3307 - J461.53$.

Thus $P = 3307\ \omega$

$Q = 461.53\ \omega$ } leading

$pf = \cos\theta$

$= \cos[\tan^{-1}(\frac{Q}{P})] = 0.99$ lead.

Q. The voltage of a ckt is $10\angle 15°$ & current is $2\angle -45°$. The active & reactive powers -?

$V = 10\angle 15°$ $\qquad \theta = 60°$ (lag) $\quad$ I lags V by 60°.

$I = 2\angle -45°$

$S = V I^*$

$= 10 \times 2 \angle 60°$

Active power $= 20 \cos 60° + 20 \sin 60°$

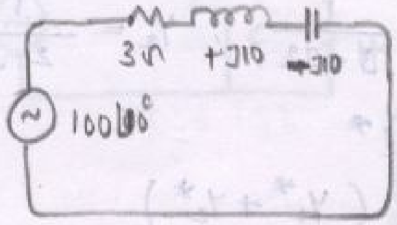
$=$

Q. The current through the current coil of a watt meter $I = 1 + 2\sin\omega t$, & the voltage across the pressure coil $v = 2 + 5\sin 3\omega t$. then the wattmeter reading is -?

wattmeter reading $= 1 \times 2 = 2\omega$.

[corr. to the funda. of current, the voltage is not present and corr. to the 3rd har. of voltage, the current is not present.]

Q. The reactive power drawn from the source — ?



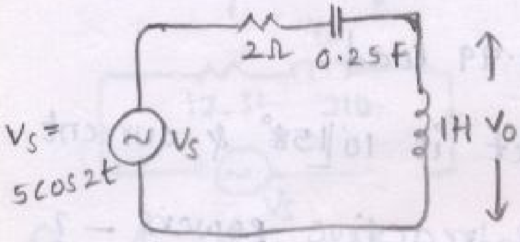
→ Lagging VAR = leading VAR

$\because X_L = X_C$.

Net reactive power =

Model – 7 : AC series & parallel circuits :-

Q. find V_0 :



$\omega = 2$,

$X_c = \frac{1}{\omega C} = 2\Omega$

$X_L = \omega L = 2 \times 1 = 2$

$X_L = X_c$

$\Rightarrow Z = R.$

$i(t) = \frac{V_s}{R} = 2.5 \cos 2t$.

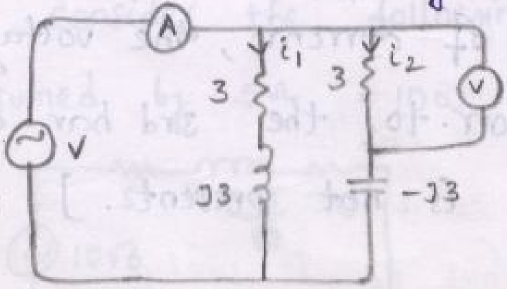
$V_0 = V_L(t) = L \cdot \frac{di(t)}{dt} = -5 \sin 2t$

Q. In ac ckt having RL&C and operating at lagging pf. Increase in freq. will result

— reduce current.

$\because \uparrow X_L = 2\pi f L \rightarrow$ reactance increases so current further decreases.

Q. In the ckt. shown, the voltmeter indicates 30V. The reading of ammeter will be — ?



$i_2 = \frac{30}{3} = 10\,A$

$Z_2 = 3 - j3 \rightarrow 45°$ lead.

$I_2 = 10 \angle 45°$

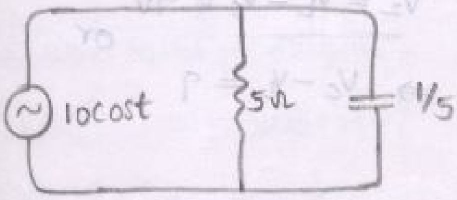
$V = i_2 Z_2 = 10\sqrt{9+9}$

$= 30\sqrt{2}\ \angle \tan^{-1}\left(\frac{-3}{3}\right)$

$i_1 = \frac{V}{Z_1} = 10 \angle -45^\circ$

$\rightarrow I = 10 \angle -45^\circ + 10 \angle 45^\circ = 14.14\ A.$

Q. The current I drawn from the V_s is — ?



$X_C = \frac{1}{1 \times \frac{1}{5}} = 5\Omega$

$Y_1 = \frac{1}{5}, \quad Y_2 = j\frac{1}{5}$

$|Y| = \sqrt{\frac{1}{25} + \frac{1}{25}} = 0.28 \angle +45^\circ$

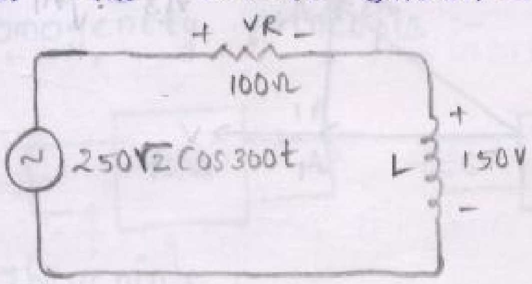
$V_s = 10 \angle 0^\circ$

$I = 10 \times 0.28 \angle 45^\circ = 2.8 \angle 45^\circ$

$\Rightarrow I = 2\sqrt{2} \angle 45^\circ$

→ power should be calculated with rms value.

Q which of the following statements are true in the circuit shown —.



$\times V_R = 100\sqrt{2}$

$\checkmark\ i = 2A$

$\checkmark\ L = 0.25\ H$

$V_r = \sqrt{V_s^2 - V_l^2}$ $\quad V_{rms} = 250$

$= \sqrt{250^2 - 150^2}$ $\quad i = \frac{V_R}{R} = \frac{200}{100} = 2A$

$= 200V$ $\quad |Z| = \frac{V}{I} = \frac{250}{2} = 125\ \Omega$

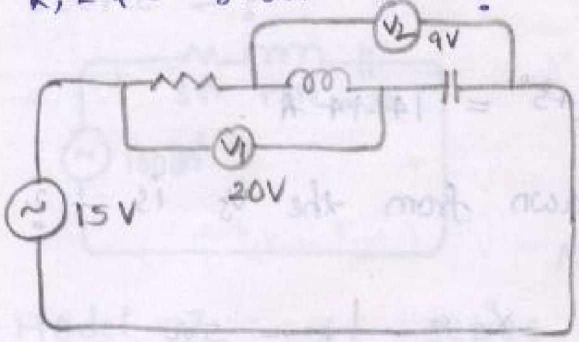
$X_L^2 = 125^2 - 100^2$

$= 75\Omega$

$75 = 300 \times L$

$\rightarrow L = 0.25\ H$

Q. for the given ckt, the voltage across R, L & C should be – ?



$V_1 = 20V$

$= \sqrt{V_R^2 + V_L^2}$

$\Rightarrow V_R^2 + V_L^2 = 20^2$

$V_2 = V_L - V_C = 9V$ or

$\Rightarrow V_C - V_L = 9$

$V_S^2 = V_R^2 + (V_L - V_C)^2 = 15^2$

$\Rightarrow 20^2 - V_L^2 + (V_L - V_C)^2 = 15^2$

$\Rightarrow V_L^2 = 20^2 - 15^2 + 9^2$

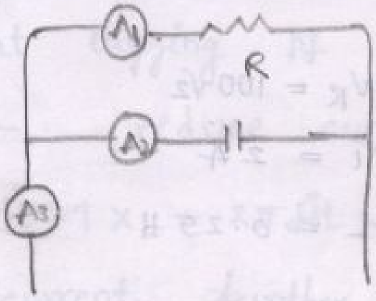
$\Rightarrow V_L = 16V$

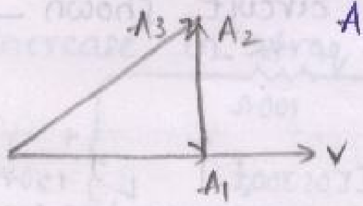
$\Rightarrow V_C = 16 - 9 = 7V.$

$V_R^2 + 9^2 = 15^2$

$\Rightarrow V_R = 12V.$

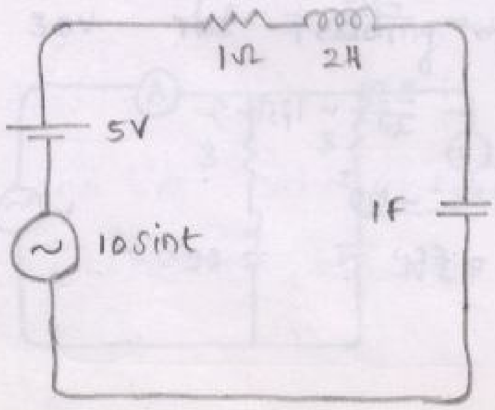
Q. In the fig. shown, A_1, A_2, A_3 are ideal ammeters. If A_1, A_3 read 5 & 13 A res. then reading of A_2 = ?





$A_3 = \sqrt{A_1^2 + A_2^2}$

Q In the fig. shown ,



4

24-06-07

Network Theorems

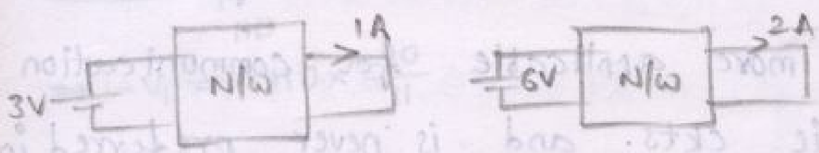
Super position theorem :-

→ Applied for linear, bilateral n/ω. only.

→ Atleast 2 ind. sources are required.

→ can be applied for V or I but not for the power.

Homogenity principle :-



Thevenin's :

→ Applicable for unilateral, bilateral, active & passive and it can't be applied for non-linear n/ω.

→ Thevenin's & Norton's eq. ckts are applicable only for practical voltage & current sources res. and not for ~~ind.~~ ideal sources.

Q Maximum power transfer theorem :-

Case 1: The power transfered to the load is max, if both the source and load impedance are pure resistive $\Rightarrow R_L = R_s$.

$$P_{max} = \frac{V^2}{4R}$$

condi. for max. p. transfer

Case 2: The source is complex impedance, but the load is pure resistive,

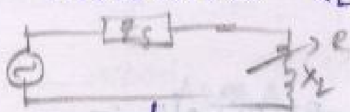
condi. for max. power transfer = $R_L = |Z_s|$

$$\Rightarrow R_L = \sqrt{R_s^2 + X_s^2}$$

Case 3:- Both the source and load has complex impedance -

condi. for max p transfer: $Z_L = Z_s^*$ or $Z_s = Z_L^*$

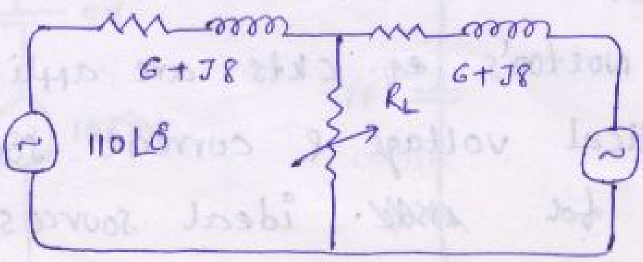
Case 4: Source has complex impedance, but load has variable resistance R_L, but fixed reactance X_L.



condi. for max. p. transfer = $R_L = \sqrt{R_s^2 + (X_L + X_s)^2}$

→ MPT is more applicable for communication and electronic ckts. and is never preferred in power systems owing to 50% voltage drop and 50% losses in the source impedance.

Q. Determine max. power dissipated through R_L.



$V_{Th} = 100V \quad Z_{Th} = 5\angle 53.13^\circ$

condi. for MPT $\rightarrow R_L = |Z_{Th}| = 5$

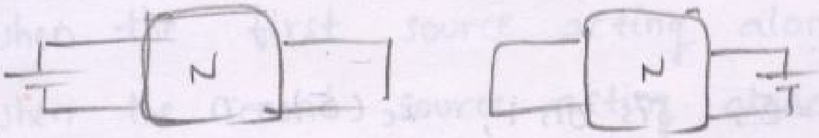
$I = \dfrac{100\angle 0^\circ}{5+3+j4} = 11.13\angle{-26.5^\circ}$

$\Rightarrow P_{max} = I^2 R = 11.13^2 \times 5 = 625W$.

Reciprocity Theorem:-

→ A n/w which satisfies the reciprocity Th. is known as reciprocal n/w.

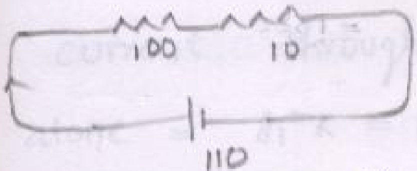
→ The n/w N should not contain any active elements or control sources.



→ If $R_1 : R_2 = 1:4$ ~~$R_1:R_2 = 4:1$~~

$\Rightarrow V_1 : V_2 = 1:4$ then $V_1 = V \times \frac{1}{5}$

Q



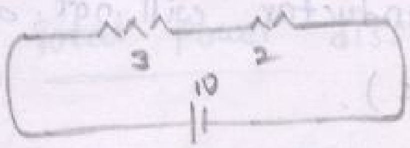
$\rightarrow R_1 : R_2 = 10:1$

$V_1 : V_2 = 10:1$

$\Rightarrow V_1 = 110 \times \frac{10}{11}$

$V_2 = 110 \times \frac{1}{11}$

Q.



$R_1 : R_2 = 3:2$

$V_1 : V_2 = 3:2$

$\Rightarrow V_1 = 10 \times \frac{3}{5}$; $V_2 = 10 \times \frac{2}{5}$

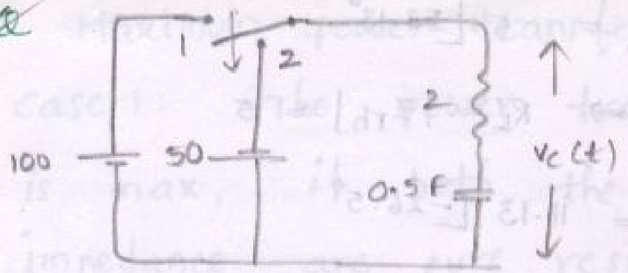
Transients

→ charging function $f(t)^+ = Ae^{-t/\tau} + B$

Discharging function $f(t)^- = A.e^{-t/\tau}$

where $A = f(t=0^+) - f(t=\infty)$

and $B = f(t=\infty)$

Q 

The switch S is on pos. 1 for a long time. At $t=0$, S is thrown onto pos. 2 find $v_c(t)$, for $t>0$.

At $t=0^-$, S is on 1, $\Rightarrow v_c(t=0^-) = 100\,V$

At $t=0^+$, S is on 2, $\Rightarrow v_c(t=0^+) = 100V$.

(since the voltage across the capacitor can't change instantaneously.

At $t=\infty$, S is on 2 $\Rightarrow v_c(t=\infty) = -50V$

(It discharges 100V and charges upto $-50V$).

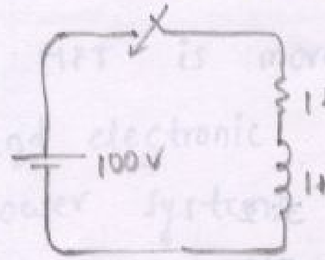
At $t=0^-$, S is on 1, $i_c(0^-) = 0$

$A = 100-(-50) = 150$

$B = -50V$

$v_c(t) = Ae^{-t/\tau} + B$ $\qquad \tau = RC = 1\,sec$

$= 150e^{-t} - 50$

Q 

At $t=0^-$, S is open $i(t) = 0$

At $t=0^+$, S closed $i(t) = 0$

(since inductor will not allow the current to change).

At $t=\infty$, S closed $\Rightarrow i(t) = 100\,A$

$\therefore A = 0-100 = -100$

$B = 100$

$i(t) = 100 - 100e^{-t/\tau}$

At $t = 0^-$, (S opened) $v(t) = 0\,V$

At $t = 0^+$, (S closed) $v(t) = 100\,V$

At $t = \infty$, (S closed) $v(t) = 0\,V$

for inductor : charging fun : current
discharging fun : voltage

for capacitor : charging fun : voltage
discharging fun : current

Q. The n/w consists of 2 ideal sources and several resistances, one of which is R. The power consumed by the resistor is P_1 watts when the first source acting alone & P_2 W, when the second source acting alone. If both the sources are acting 2gether then the power dissipated by R is —.

current through R for 1st source acting alone = $I_1^2 R = P_1 \Rightarrow I_1 = \sqrt{\frac{P_1}{R}}$

$|||^{ly}$ $I''^2 R = P_2'' \Rightarrow I'' = \sqrt{\frac{P_2}{R}}$

when both are acting together, $I = I_1 + I''$

Total power dissipated $= I^2 R$

$$= \left[\sqrt{\frac{P_1}{R}} \pm \sqrt{\frac{P_2}{R}}\right]^2 \cdot R$$

$$\Rightarrow P_{eq} = (\sqrt{P_1} \pm \sqrt{P_2})^2$$

A linearity — 1. Superposition

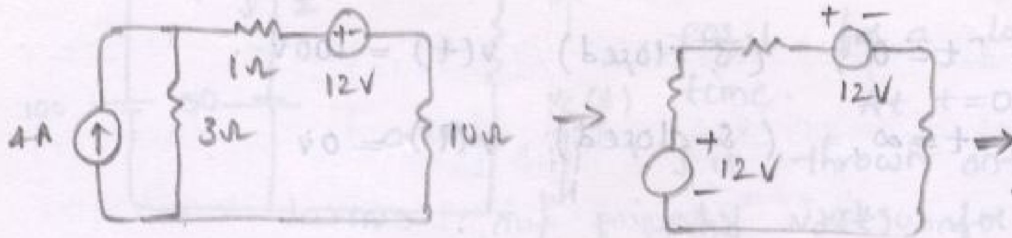
B structure — 2. Norton's

C eq. ckt — 3. Telligen

D Bilateral — 4. Millman

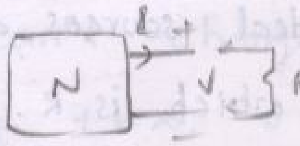
5. Reciprocity

Q find the current,



$\Rightarrow 0A$

Q. The v–I relation : $V = 3I + 4$ if now, a resistor of 1Ω is connected shown. then the value of $I = ?$



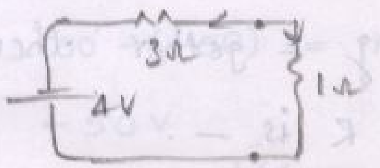
$V = 4 + 3I$ If $I = 0$, $V = 4V$

$\Rightarrow V_{Th} = 4V$

If $V = 0 \Rightarrow I_N = (-)\frac{4}{3}$ A
dire.

$\therefore R_{Th} = \frac{4}{4/3} = 3\Omega$

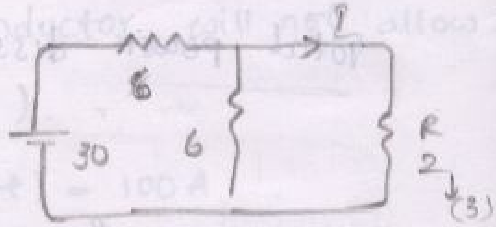
thus



$I = -1A$

→ for applying Thevenin's the n/w other than load should be linear.

Q. In the ckt, the current in 2Ω (R) is 3A, if R is changed to 3Ω, the new value of I will be – ?

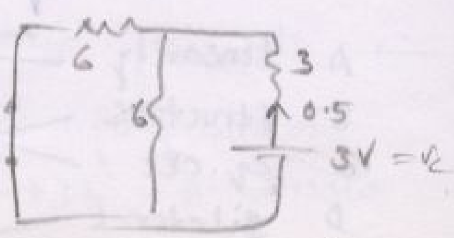


$I = 4A$

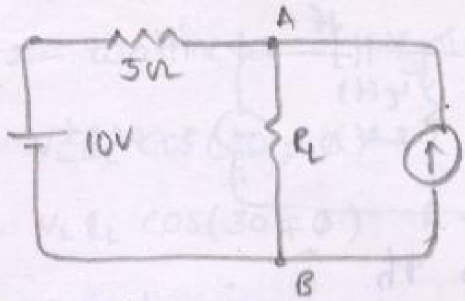
using compensation th.

$V_c = I \times \Delta R = 3 \times 1 = 3V$

The new value of $i = 2.5$ A



Q. find the Thevinin's eq. ckt.

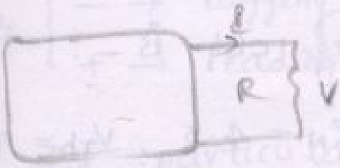


$V_{Th} = 20V$

$R_{Th} = 5\,\Omega$

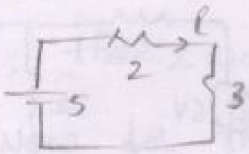
Q. for the n/w shown, when $R=\infty$, $V=5^V$

$R=0$, $I = 2.5\,A$ then $R = 3\,\Omega$, $I = ?$

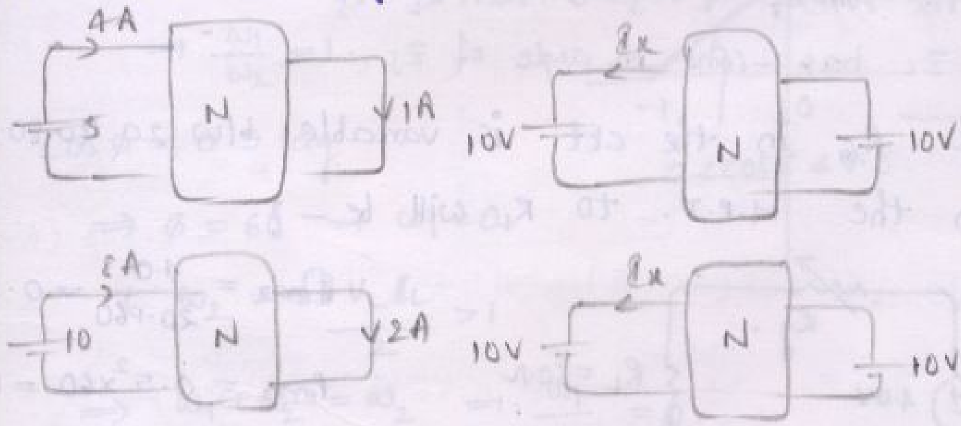


$R_{Th} = \frac{5}{2.5} = 2\,\Omega$

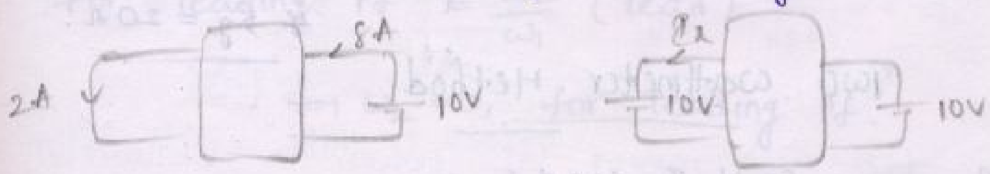
$I = \frac{5}{5} = 1A.$



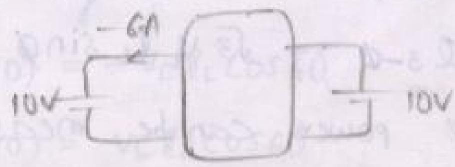
Q. The n/w 'N' of fig(a) & fig(b) is passive and contains only linear resistors. The port i's in fig (a) are marked. using these values, find I_x in fig (b).



$\Rightarrow I_x' = -8A$; apply reciprocity

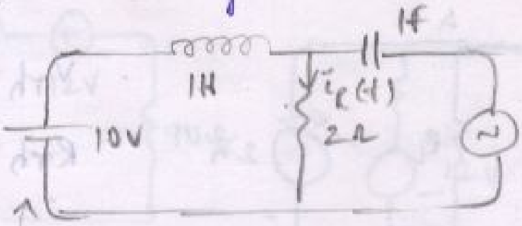


$\Rightarrow I_x'' = 2A$



$I_x = -8+2 = -6A$

Q. find $i_R(t)$ through the resistor, under s.s. condi.s

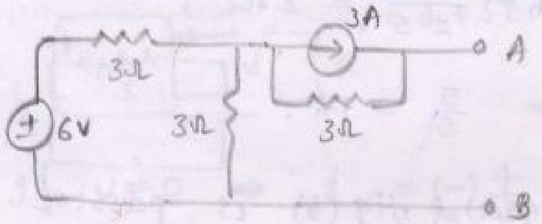


By using superposition th.

$i_R(t) = 5 + 3.16 \angle 18.44$

$= 5 + 3.16 \cos(2t + 18.44)$.

Q. for the ckt shown, Norton's eq. ckt. is –.



$R_N = 4.5$, $V_{Th} = 9V$

$I_N = \frac{9}{4.5} = 2A$

Q. In M.P.T., from ac source to variable load.

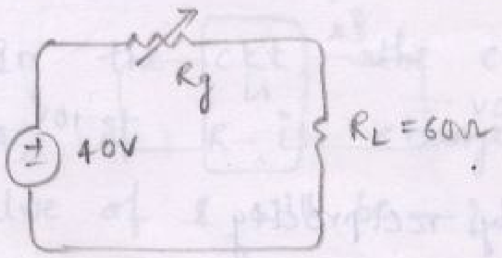
(1). $Z_L \to$ inductive, if Z_s inductive

(2). $Z_s + Z_L = 0$

(3). The sum of $X_L + X_s = 0$ then $R_L = R_s$

(4). Z_L has same ph. angle of Z_s.

Q. If R_G in the ckt is variable b/w 20 to 80 Ω then the M.P.T. to R_L will be – ?



$I_{max} = \frac{40}{20+60} = 0.5A$

$P_{max} = 0.5^2 \times 60 = 15W$

[I is max, when R_{total} is min]
ie $R_g = 20\,Ω$.

Two wattmeter Method

$P_{3-\phi} = \sqrt{3}\, V_L I_L \cos\phi$

$Q_{3-\phi} = \sqrt{3}\, V_L I_L \sin\phi$

$3-\phi$ power can be measured by

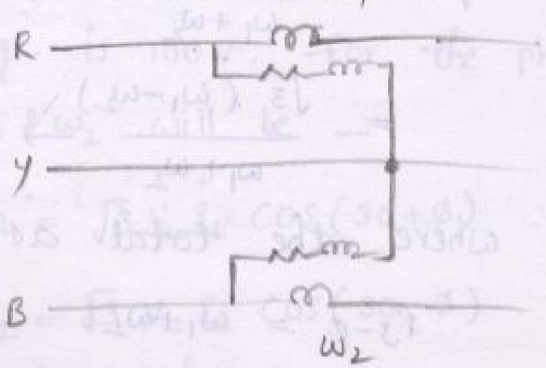
(1). 3 - wattmeter [can be used only for Y)

(2). 1 – wattmeter [balanced Y]

(3). 2 – wattmeter [Y or Δ, balanced or unbalanced]

$\omega_1 = V_L I_L \cos(30 \pm \phi)$

$\omega_2 = V_L I_L \cos(30 \mp \phi)$

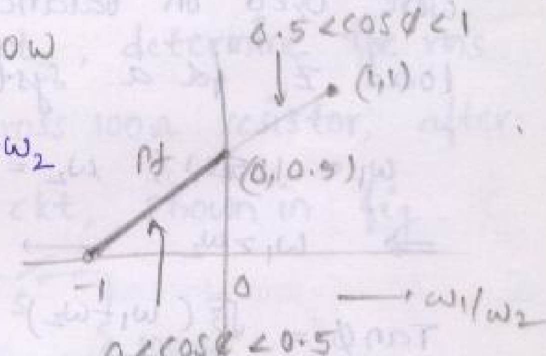


$\omega_1 \begin{cases} + \rightarrow \text{lagging} \\ - \rightarrow \text{leading} \end{cases}$

$\omega_2 \begin{cases} - \rightarrow \text{laggeng} \\ + \rightarrow \text{leading} \end{cases}$

Q. for particular pf of the load, the 2 wattmeter ω_1 & ω_2 read as, 200 & 100 W res. Now for the same power, factor at lead, the wattmeter reading — 2

$\omega_1 = 200\,W \quad \omega_2 = 100\,W$



1). $\cos\phi = 1 \rightarrow \phi = 0^\circ \Rightarrow \omega_1 = \omega_2$

$\rightarrow \frac{\omega_1}{\omega_2} = 1.$

2). $\cos\phi = 0.5$ lag

$\Rightarrow \phi = 60^\circ \rightarrow \omega_1 = 0.$

& $\omega_2 = \frac{\sqrt{3} V_L I_L}{2} > 1$

$\Rightarrow \omega_1 + \omega_2 = \omega_2 \rightarrow \frac{\omega_1}{\omega_2} = 0.$

for leading pf $= \frac{\omega_2}{\omega_1}$ (lead)

$\rightarrow \omega_2 = 0$, for leading pf.

3). $\cos\phi = 0 \rightarrow \phi = 90^\circ$ for lag

$\Rightarrow \omega_1 = V_L I_L \cos(30+90) = -V_L I_L \cos 60^\circ$

$\omega_2 = V_L I_L \cos(30-90) = V_L I_L \cos 60^\circ$

$\Rightarrow \omega_2 = -\omega_1 \Rightarrow \omega_1/\omega_2 = -1$

Thus for W_1, showing $-ve \rightarrow$ zpf lag.

W_2 showing $+ve \rightarrow$ zpf lead.

$$\tan\phi = \frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2} \longrightarrow \text{for lagging pf}$$

$$= \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} \longrightarrow \text{for leading pf.}$$

where the total active power,

$$P_{3-\phi} = W_1 + W_2$$

$$\& \quad Q_{3-\phi} = \sqrt{3}(W_1 - W_2)$$

Q. Readings of 2 wattmeter, $W_1 = 1154W$, $W_2 = 577W$ were obtained when the 2 wattmeter method were used on balanced load. The Δ-connected load Z for a system volt. of 100V will be —.

$W_1 = 1154W$, $W_2 = 577W$

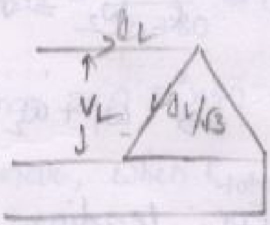
$\Rightarrow W_1 > W_2 \longrightarrow$ pf $(0.5 \rightarrow 1)$

$$\tan\phi = \frac{\sqrt{3}(W_1 - W_2)}{W_1 + W_2} = \frac{1}{\sqrt{3}}, \quad \cos\phi = \frac{\sqrt{3}}{2}$$

$$= 0.866 \text{ (lead)}$$

$\therefore \sqrt{3} V_L I_L \cos(30 - \phi) = 1154$

$$\Rightarrow I_L = \frac{1154}{\sqrt{3} \times 100 \times 1} = 6.66 \text{ A}$$



$$\Rightarrow Z = \frac{V_{ph}}{I_{ph}} = \frac{\sqrt{3} V}{I} = \frac{\sqrt{3} \times 100}{6.66} = 26$$

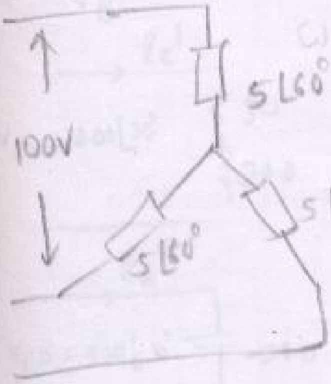
$$= 26 \angle -30^\circ$$

Q. In the measurement of power on balanced load, by 2 wattmeter in a $3-\phi$ ckt. are $W_1 = 3kW$, $W_2 = 1kW$ res. The later being obtained after reversing the connections to the current coil. The pf of the load — ?

$\omega_1 = 3\,kW$; $\omega_2 = 1\,kW$.

$\Rightarrow \tan\phi = \dfrac{\sqrt{3}\,(4)}{2} \rightarrow \cos\phi = 0.271$ lead

Q The L-L ~~i/p~~ voltage to the 3-ϕ 50 Hz ac. ckt shown in fig. is 100 V. for the ph. seq. RYB the ω_1 & ω_2 will be —.



$$\omega_1 = \sqrt{3}\, V_L I_L \cos(30+\phi)$$

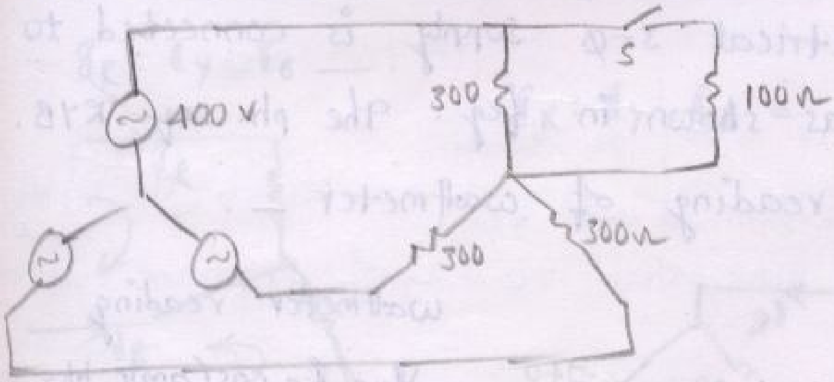
$$\omega_2 = \sqrt{3}\, V_L I_L \cos(30-\phi)$$

$$I_{1-\phi} = \frac{100}{\sqrt{3}} / 5 = 11.54\,A = I_L$$

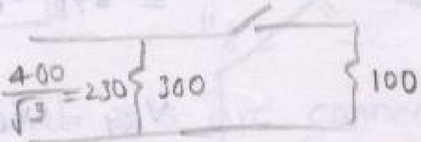
$$\Rightarrow \omega_1 = \sqrt{3} \times 100 \times 11.54 \times \cos(30+60)$$

$$\omega_2 = \sqrt{3} \times 100 \times 11.54 \times \cos(30-60)$$

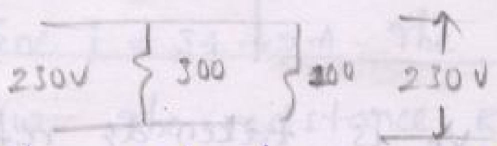
Q using Thevenin's eq. ckt, determine the rms value of the voltage across 100Ω resistor, after the 'S' is closed in 3-ϕ ckt, shown in fig.



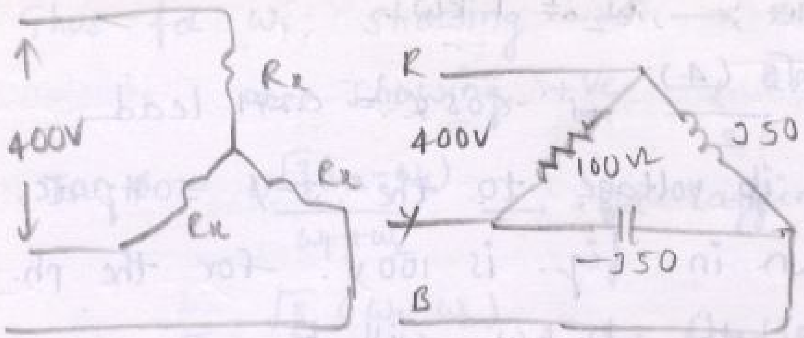
1-ϕ load,



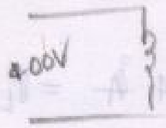
when the switch is closed



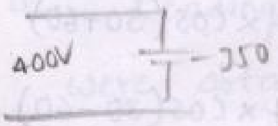
Q A set of 3 equal R's each of R_x connected in Y across RYB,



$$P'_{1-\phi} = \frac{400^2}{100} = 1600\,W$$

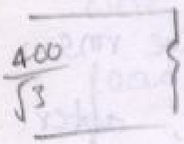


$$P_{consumed} = P''_{1-\phi} = 0$$



$$P_{consumed} = P'''_{1-\phi} = 0.$$

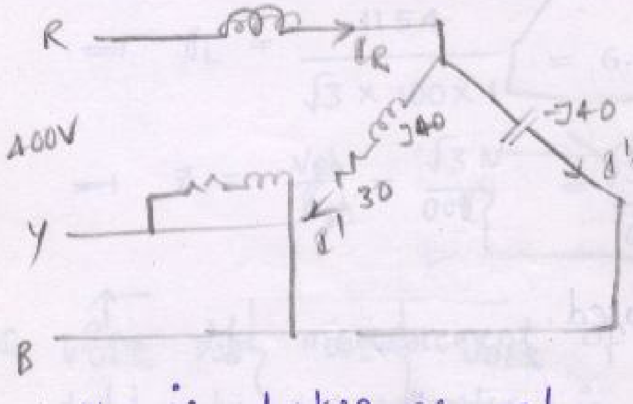
Total power drawn = 1600W.



$$P_{1-\phi} = \frac{400^2}{3\,R_x}$$

$$P_{3-\phi} = \frac{400^2}{R_x} = 1600 \Rightarrow R_x = 100\,\Omega.$$

Q A symmetrical 3-φ supply is connected to the n/w as shown in fig. The ph. seq. RYB. find the reading of wattmeter.



wattmeter reading $= V_{YB}\, I_R \cos(\text{angle b/w } V_{YB}, I_R)$

$V_{RY} = V_R - V_Y$

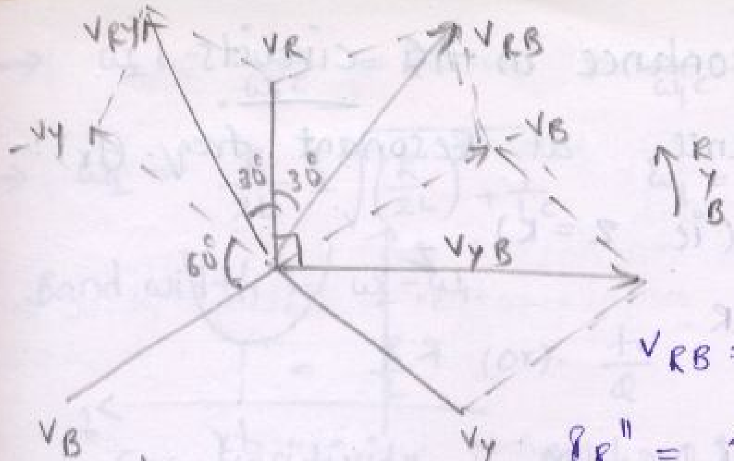
$= V_R + (-V_Y)$

$V_{YB} = V_Y + (-V_B)$

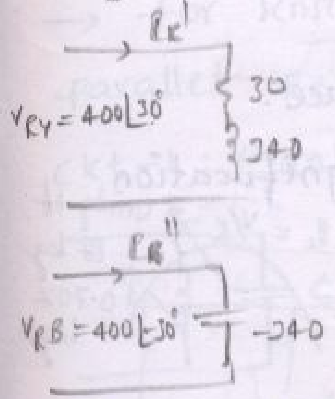
V_R is taken as ref.

V_{RY} leads V_R by 30.

$$\Rightarrow I_R' = \frac{400\angle 30}{50\angle 53.13} = 8\angle -23.13^\circ \rightarrow \text{lag } V_R \text{ by } 23.13^\circ.$$



$$V_{RB} = V_R - V_B = V_R + (-V_B)$$



$$I_R'' = \frac{400\angle -30^\circ}{-J40} = 10\angle 60^\circ$$

$$\therefore \; I_R = I_R' + I_R''$$

$$= 13.47\angle 23.82^\circ$$

So angle b/w V_R, I_R is 23.82°.

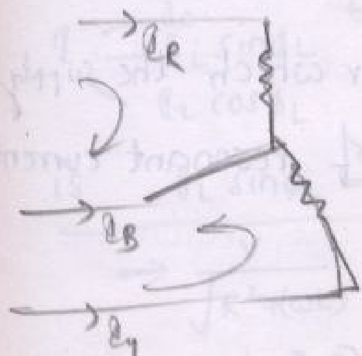
Angle of V_{YB} w.r.t V_R is 90°.

Thus, the angle of I_R w.r.t. $V_{YB} = 90 + 23.82$
$= 113.82^\circ$.

Thus wattmeter reading $= 400 \times 13.47 \times \cos(\downarrow)$

Q. for the ckt shown, the ratio of currents $I_R : I_Y : I_B$ ___.

$I_R : I_Y : I_B = 1 : 1 : \sqrt{3}$.



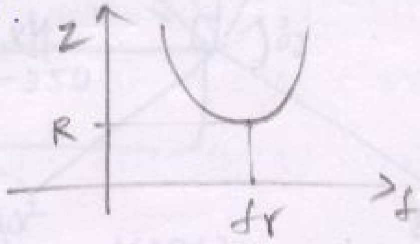
Q 3 identical Z's are connected in across 3-φ supply of 400V. The line $i = 34.65$ A. The total power is 14.4 kW. The resistance, R of the load in each ph. in Ω is ___. 16 Ω

$$400 \Big/ \frac{34.65}{\sqrt{3}}$$

Resonance in AC circuits

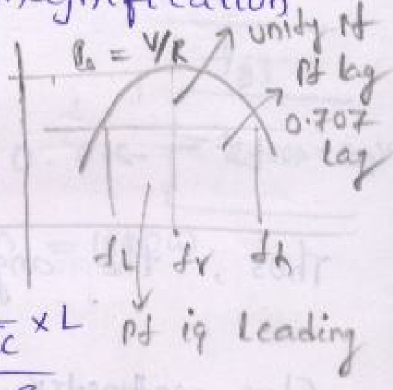
Series resonance, at Resonant freq. (f_r) :-

1. Z is min (ie $Z = R$).
2. $V_S = V_R = I_0 R$.
3. $pf = 1$
4. Acceptor ckt. $I_0 = \frac{V}{R}$



5. voltage multiplication takes place.
6. Q = amplification factor = magnification factor = Quality factor

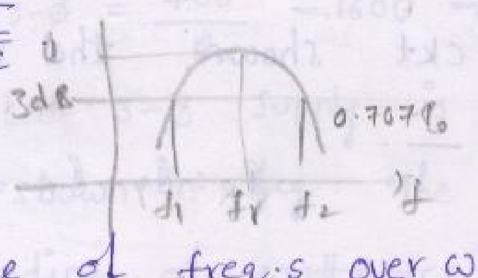
$$= \frac{V_L}{V_S} = \frac{V_C}{V_S}$$

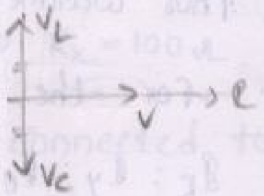


7. $f_r = \frac{1}{2\pi\sqrt{LC}}$, $X_L = X_C$.

$$Q = \frac{V_L}{V} = \frac{I_0 X_L}{I_0 R} = \frac{\omega_0 L}{R} = \frac{\frac{1}{\sqrt{LC}} \times L}{R}$$

$$\Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$





Band width :-

The range of freq.s over which the supply current is more than 70.7% of resonant current.

At f_1 & f_2

$$I = \frac{I_0}{\sqrt{2}}$$

$$\frac{V}{Z} = \frac{V}{\sqrt{2}R} \Rightarrow Z = \sqrt{2}R$$

$$R^2 + X^2 = 2R^2$$

$$\Rightarrow X = R \longrightarrow \text{Net reactance} = \text{resistance}$$

$$\Rightarrow X_L - X_C = R \quad (\text{or}) \quad X_C - X_L = R$$

$$\Rightarrow \frac{-1}{2\pi f_2 C} + 2\pi f_2 L = R \qquad \frac{1}{2\pi f_1 C} - 2\pi f_1 L = R$$

$\Rightarrow \omega_2 L - \frac{1}{\omega_2 C} = R$ $\qquad \frac{1}{\omega_1 C} - \omega_1 L = R$

$\Rightarrow \omega_2 = \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$ $\qquad \omega_1 = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$

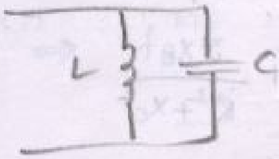
Band width $= \omega_2 - \omega_1$

$= \frac{R}{L}$ (or). $\frac{f_0}{Q}$

→ For sensitivity ↑, BW ↓, Q ↑

parallel Resonance :-

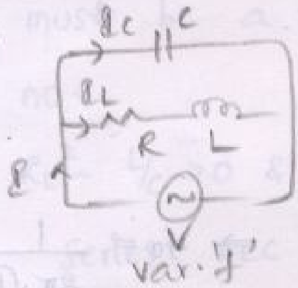
ckt-1 : (Tank ckt) : imaginary $(Y_{total}) = 0$

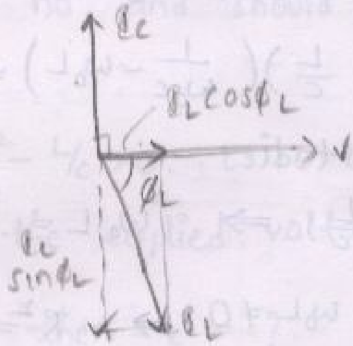


At resonance $\omega C - \frac{1}{\omega L} = 0$

$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$

ckt-2 :





$\phi_L = \tan^{-1}\left(\frac{X_L}{R}\right) \simeq 90°$

I — I_C — $I_L \sin\phi_L$ — $I_L \cos\phi_L$ $\qquad$ suppose, if $I_c = I_L \sin\phi_L$

$\therefore$ condi. for resonance of the ckt.

is $I_L \sin\phi_L = I_C$

$\Rightarrow \frac{V}{\sqrt{R^2+(\omega L)^2}} \cdot \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}} = V \cdot \omega_C$

R is small, so R^2 is neglected.

then $\Rightarrow \frac{V}{\omega L} \cdot \frac{\omega L}{\omega L} = V \cdot \omega_C$

$\Rightarrow \omega^2 = \frac{1}{LC} \rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$

$\because \phi_L \simeq 90°, \Rightarrow \cos\phi_L \simeq 0$

$\rightarrow I = I_L \cos\phi_L \simeq 0$

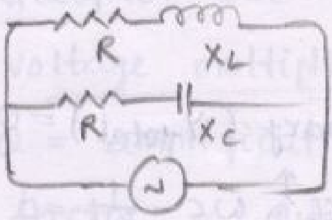
Thus min. current drawn by the ckt.

→ I is small so Rejector ckt.

→ Z is very high.

→ so current magnification is taking place in the ckt.

ckt – 3:



At Resonance,

img (Y total) = 0

$$Y_1 = \frac{R}{R^2+X_L^2} - j\frac{X_L}{R^2+X_L^2}$$

$$Y_2 = \frac{R}{R^2+X_C^2} + \frac{jX_C}{R^2+X_C^2}$$

$$\Rightarrow \frac{X_C}{R^2+X_C^2} = \frac{X_L}{R^2+X_L^2}$$

$$\Rightarrow \left(R^2 - \frac{L}{C}\right)\left(\frac{1}{\omega_0 C} - \omega_0 L\right) = 0$$

case studies :-

1. $R^2 \neq \frac{L}{C} \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$

2. $\frac{1}{\omega_0 C} - \omega_0 L \neq 0 \Rightarrow R^2 = \frac{L}{C}$

$$\Rightarrow R = \sqrt{L/C}$$

ie img (Y total) = 0 → ckt resonates at all the frequencies.

The total admittance of ckt at resonance,

$$Y_T = R.P[\, Y_{total} \,]$$

$$= \frac{R}{R^2+X_L^2} + \frac{R}{R^2+X_C^2}$$

3. If $R^2 = \frac{L}{C}$ & $\frac{1}{\omega_0 C} = \omega_0 L$

→ ckt resonates at all the freq.s.

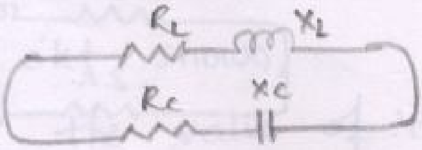
If $R^2 = \frac{L}{C} \Rightarrow Y_T =$

If $\frac{1}{\omega_0 C} - \omega_0 L = 0 \rightarrow Y_T = \frac{2R}{R^2 + \frac{L}{C}}$

if $R^2 = \frac{L}{C}$ & $\frac{1}{\omega_0 C} - \omega_0 L = 0 \Rightarrow Y_T = \frac{1}{R}$.

$\rightarrow$ current at the resonance $= V \cdot Y_T$.

Case 4:



At Resonance, img (Y-total) = 0

$$\Rightarrow \frac{R_C}{R_C^2 + X_C^2} = \frac{R_L}{R_L^2 + X_L^2}$$

$$\Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}\left[\sqrt{\frac{R_L^2 - L/C}{R_C^2 - L/C}}\right]$$

f_0 must be a real no. and should not be a complex no.

$\Rightarrow R_L^2 - L/C > 0$ & $R_C^2 - L/C > 0$.

Q. In a series RLC, the applied voltage 200V, $R = 10\Omega$, $X_L = X_C = 20\Omega$ then $V_C = ?$

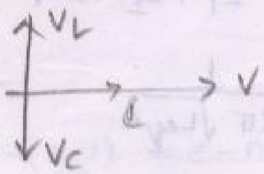
$$Q = \frac{I_0 X_L}{I_0 R} = \frac{20}{10} = 2$$

$\Rightarrow |V_L| = |V_C| = Q V_S$

$= 2 \times 200 = 400V$.

$V_C = 400 \angle -90^\circ$

$V_L = 400 \angle 90^\circ$



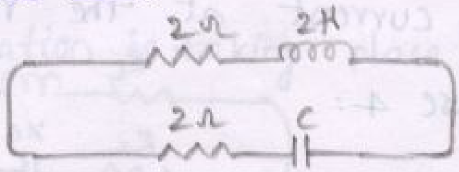
Q. In a series RLC, $Q = 100$. If all the compo.s are doubled then $Q' - ?$

$$Q = \frac{1}{R}\sqrt{L/C}$$

$$Q' = \frac{1}{2R}\sqrt{\frac{2L}{2C}} = \frac{Q}{2} = 50.$$

Q. In series RLC, pf at f_1 <u>0.707 lead</u> and at f_2 <u>0.707 lag</u> and at f_0 <u>1 (upf)</u>

Q. Find C, at resonance —.

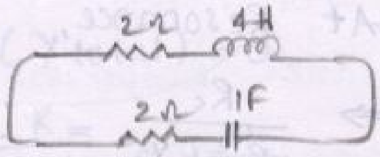


$(X_L = R_C)$

$R^2 = \frac{L}{C}$

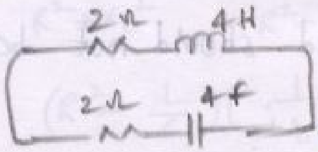
$\Rightarrow C = \frac{2}{4} = 0.5\,F$.

Q. Find Resonant freq — ?



$f_0 = \frac{1}{2\pi\sqrt{4}} = \frac{1}{4\pi}$

Q. Determine the current supplied by the source. If the ckt is at resonance —.



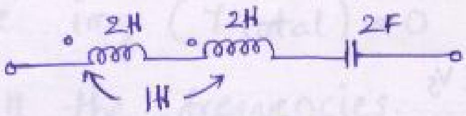
$V = 10\angle 20^\circ$.

$\rightarrow \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{4}$

$Y_{total} = \frac{2R}{R^2 + L/C} = \frac{4}{5}\,\mho$

$I = VY = 10\angle 20^\circ \times \frac{4}{5} = 8\angle 20^\circ$.

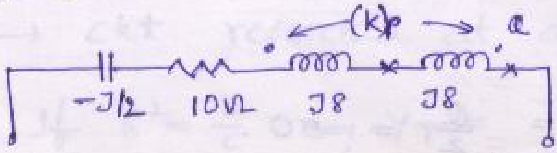
Q. Find Resonant freq —.



$L_{eq} = L_1 + L_2 + 2M$

$f_0 = \frac{1}{2\pi\sqrt{L_{eq}C}}$

Q. In the ckt shown V & I are in phase (resonance) the value of k and the polarity of coil PQ are —



$X_L = X_C$

$\Rightarrow X_L = X_{L_1} + X_{L_2} \pm 2X_m$

$\Rightarrow X_L = X_{L_1} + X_{L_2} - 2X_m$

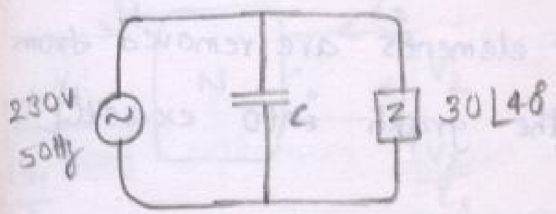
2). $X_L = X_C = 12 = 8 + 8 - 2X_m$

$\Rightarrow X_m = 2\,\Omega$

$M = k\sqrt{L_1 L_2} \Rightarrow k = \dfrac{M}{\sqrt{L_1 L_2}} = \dfrac{\omega M}{\sqrt{\omega L_1 \times \omega L_2}}$

$= \dfrac{2}{\sqrt{8 \times 8}} = 0.25.$

Q. In the ckt shown, what is the value of 'c' will have a UPF at ac source –



$I_L \sin\phi_L = I_C$

$\dfrac{V}{Z} \sin 40° = V \cdot \omega C$

$\Rightarrow \dfrac{1}{30} \sin 40° = \omega C$

$\Rightarrow C = \dfrac{1}{2\pi \times 50 \times 30} \sin 40°$

$= 68.1\ \mu F.$

Q. $R = 50\,\Omega$

$L = 100\ mH$

$C = 1\ \mu F$ then $f_1 = ?$

$$f_1 = \frac{1}{2\pi}\left[\frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}\right]$$

Graph theory

→ fundamental of f-loops or basic loops = no. of links. $\Rightarrow l = e - n + 1$

→ for 'e' elements:

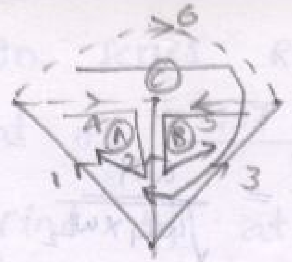
branches / twigs = $n-1$

links / chords = $e-(n-1) = e-n+1$.

f-loop incidence matrix (or) Basic Loop incidence Mat :-

$$L = \begin{array}{c} \xrightarrow{f\text{-loop}} \\ e \downarrow \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}_{e \times l} \end{array}$$

L_{ij}
- +1 if ith element is incident in jth f loops and is oriented in the same direction.
- −1 – do – in the opposite direction.
- 0 if not incident in jth f-loop



$\rightarrow$ f-loop

$$L = \begin{array}{c|ccc} & Ⓐ & Ⓑ & Ⓒ \\ \hline 1 & 1 & 0 & 1 \\ 2 & -1 & -1 & 0 \\ 3 & 0 & 1 & -1 \\ 4 & 1 & 0 & 0 \\ 5 & 0 & 1 & 0 \\ 6 & 0 & 0 & 1 \end{array}_{6\times 3}$$

$$V = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix}_{6\times 1}$$

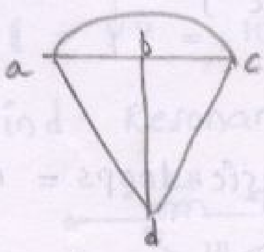
$\therefore$ n/w eq $= L^T[V] = 0.$

cut set :-

when a set of elements are removed from the graph, it divides the graph into exactly 2 unconnected sub graphs.

$\rightarrow$ no. of fundamental cutsets = no. of branches $= n-1$

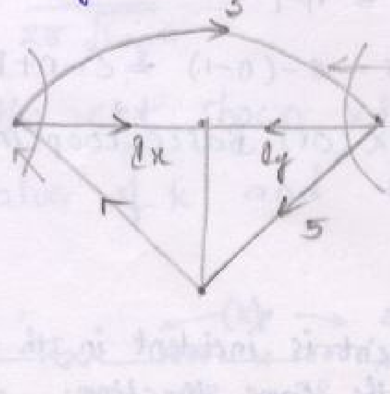
Q. Fig. below shows a graph of the n/w, A proper tree is choosen to analyse, the n/w will contain the edges.



ad, bd, cd

ab, bd, bc

Q. In the graph of the n/w, the currents are marked by arrows, the values of I_x & I_y ——.



cutsets :-

$\Rightarrow 1 - I_x - 3 = 0$

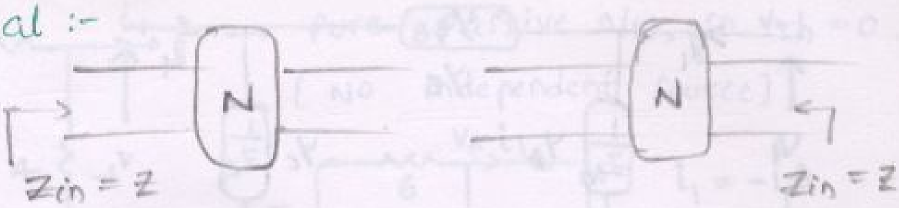
$\Rightarrow I_x = -2$

$3 - I_y - 5 = 0$

$\Rightarrow I_y = -2.$

Two port Networks :-

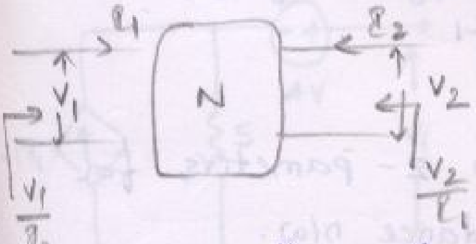
Symmetrical :-



$\Rightarrow \frac{V_1}{I_1} = \frac{V_2}{I_2} \rightarrow$ for symmetrical 2 port n/ω.

Reciprocal :-

n/ω's which obeys Reciprocity Theorem.



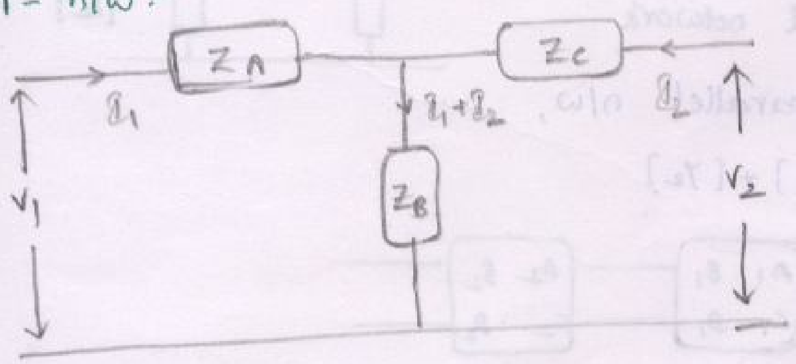
$\Rightarrow \frac{V_1}{I_2} = \frac{V_2}{I_1} \rightarrow$ for reciprocal 2 port n/ω.

→ Generally all symmetrical n/ω's are reciprocal.

→ Reciprocal n/ω's should be passive.

	condi. for Reciprocity	condi for symmetry
Z	$Z_{12} = Z_{21}$	$Z_{11} = Z_{22}$
Y	$Y_{12} = Y_{21}$	$Y_{11} = Y_{22}$
T	$AD - BC = 1$	$A = D$
h	$h_{12} = -h_{21}$	$h_{11}h_{22} - h_{12}h_{21} = 1$

T- n/ω :-



$\rightarrow Z_{11} = Z_A + Z_C$

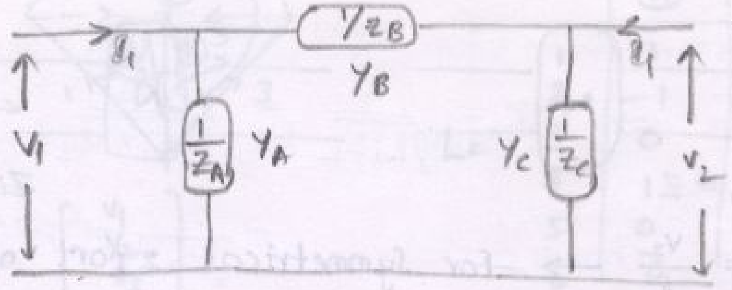
$\rightarrow Z_{12} = Z_{21} = Z_C$

$\rightarrow Z_{22} = Z_B + Z_C$

$$\therefore [Z] = \begin{bmatrix} Z_A + Z_C & Z_C \\ Z_C & Z_B + Z_C \end{bmatrix}$$

$[Y] = [Z]^{-1}$

π – n/ω :-



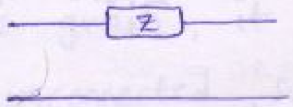
$Y_{11} = Y_A + Y_B$

$Y_{12} = Y_{21} = -Y_B$

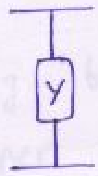
$Y_{22} = Y_B + Y_C$

$$[Y] = \begin{bmatrix} Y_A+Y_B & -Y_B \\ -Y_B & Y_B+Y_C \end{bmatrix}$$

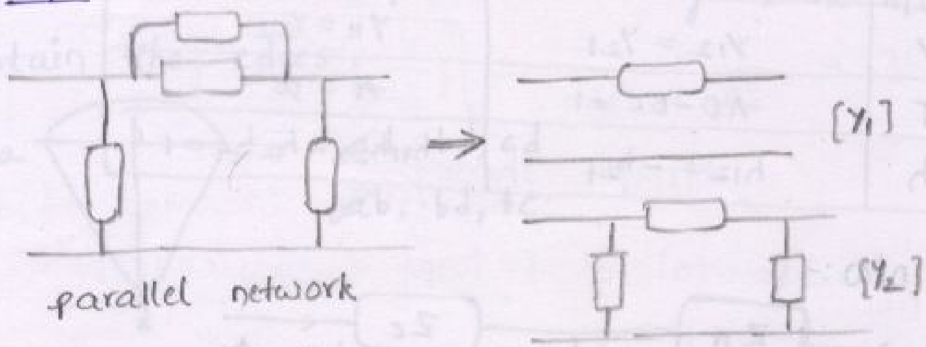
$$[Z] = [Y]^{-1}$$

⇒ 

There are no z-parameters, for series impedance n/ω.
So no z parameters for short tr. lines.

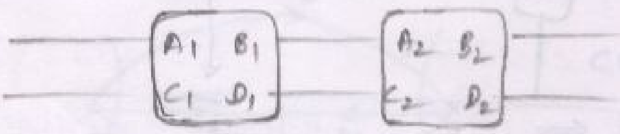
⇒ 

→ there are NO y. parameters for a shunt admittance n/ω.

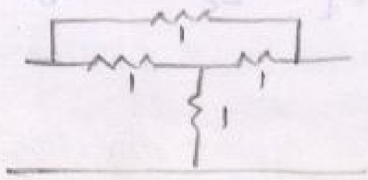
⇒ 

for a parallel n/ω,

$[Y] = [Y_1] + [Y_2]$

⇒ 

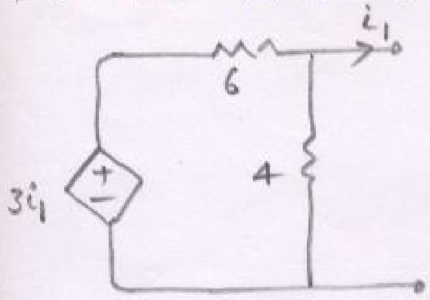
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

Q. 

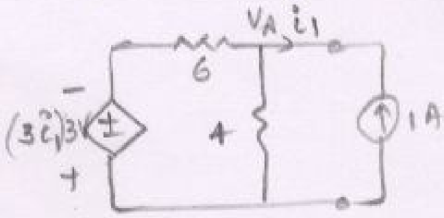
$$[Y] = [Y_1] + [Y_2] \begin{bmatrix} 5/3 & -5/3 \\ -5/3 & 5/3 \end{bmatrix}$$

control sources.

Q. find V_{Th} & R_{Th}.

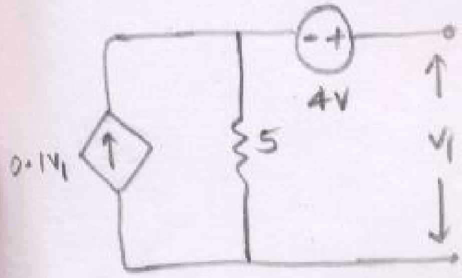


→ pure passive n/w, so $V_{Th} = 0$.
[No independent source].

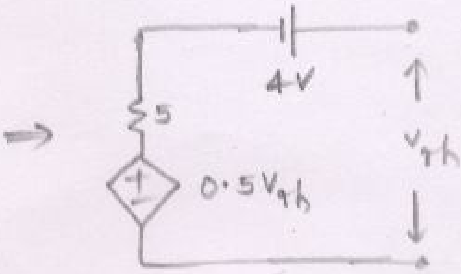


$i_1 = -1A$
$V_A = 1.2V$
$R_{Th} = \frac{1.2}{1} = 1.2$

Q. find V_{Th} & R_{Th}.



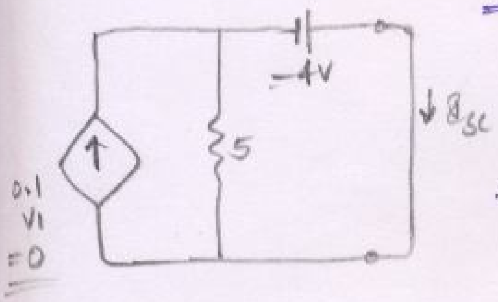
⟹



$V_1 = V_{TH}$

$V_{Th} - 4 + 5(0) - 0.5\,V_{Th} = 0$

$\Rightarrow V_{Th} = \frac{4}{0.5} = 8V$



$I_{sc} = \frac{4}{5} = 0.8A$

$\therefore R_{Th} = \frac{V_{Th}}{I_{sc}} = 10\,\Omega$